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On The Excitation Speeds of Capacitor-Excited Stand-Alone Induction Generator

A thesis submitted in partial fulfillment of M.Sc. in power systems

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2004

بسم ا الرحمن الرحيم

إِن فِي خَلْقِ السمَاوَاتِ وَالأَرْضِ وَاخْتِلاَفِ اللْيلِ وَالنهَارِ لآَياتٍ
لأُوْلِي الأَلْبابِ (**190**) الذِينَ يَذْكُرُونَ اللّهَ قِيَامًا وَقُعُودًا وَعَلَىَ
جُنُوبِهِمْ وَيَتَفَكرُونَ فِي خَلْقِ السمَاوَاتِ وَالأَرْضِ رَبنَا مَا خَلَقْتَ هَذا
بَاطِلاً سُبْحَانَكَ فَقِنَا عَذَابَ النارِ (**191**)

ǯƗمراƥ ğƏ

Acknowledge

I would like to express my deep gratitude to my supervisor Dr. Fayez Mohammed El-Sadik for his generous support and guidance through out the study period.

I am deeply indebted to the National Electricity Corporation general manager and staff for their kindly aid during these years to produce this project.

Dedication

To my merciful mother,

Brothers,

Sister,

Wife,

And Children.

To my good friends, whom I find whenever, and

wherever I need.

I dedicate this thesis.

Amir

Abstract

The principle of self-excitation, by which an induction generator can be excited by means of static capacitors connected across the machine terminals, has long been known. Although this fact was realized back in the thirties, it is only during the last few decades that the subject of the induction generator has received considerable attention. The reason is the growing emphasis on alternative energy sources, and the need for a suitable electric generator to match the requirement of operational simplicity and cost effectiveness. Although the induction generator has already emerged as an alternative choice to the synchronous generator in such areas as stand-alone wind energy farms and remote hydro-electric applications, research efforts are still directed to the understanding of self-excitation boundaries of the stand-alone unit. It is generally believed that better insight into the generation requirements of a given machine will afford the cost optimization target. The work to be presented in this thesis is concerned with the self excitation boundaries of a given machine. The experimental observations conducted show the possible existence of a pre-nominal self-excitation speed if the machine is driven at a lower-ramp rate with capacitors permanently connected. It is suggested that with the availability of user-friendly program packages such as Mathlab, algorithms for the solution of system non-linear equations could be developed for the prediction of self-excitation capacitors and speeds.

الخلاصة

منذ أمد ليس بقصير، صار مبدأ الإثارة الذاتية للمولد الحثي بواسطة توصيل مكثفات ثابتة على طرفيه معروفاً. على الرغم من أن هذه الحقيقة قد تم إدراكها منذ ثلاثينيات القرن الماضي إلا أن المولد الحثي لم يلق اهتماماً سوى في العقود الأخيرة، و السبب في ذلك زيادة الاهتمام بمصادر الطاقة البديلة و البحث عن مصدر للطاقة الكهربائية يوائم ما بين الحاجة للبساطة التشغيلية و تأثير التكلفة. مع أن المولد الحثي يظهر للعيان كبديل للمولد التزامني في مناطق التوليد المنفرد بطاقة الرياح على سبيل المثال و تطبيقات التوليد الكهرومائي إلا أن الأبحاث لا زالت تجري باتجاه محاولة فهم حدود الإثارة للمولد الحثي المنفرد. على العموم يمكن التصديق بأن فهم متطلبات التوليد للماكينة بشكل أعمق سينتج عنه تحسين مقدر في تكلفة العملية كهدف. العمل الذي سيتم عرضه في هذه الأطروحة يهتم بحدود الإثارة الذاتية لماكينة معينة. المراقبة المعملية ألقت الضوء على احتمال وجود سرعة ما تحقق الإثارة الذاتية قبل الوصول للسرعة الاسمية لهذه الماكينة إذا ما تمت قيادتها بتسارع منخفض مع وجود مكثفات موصلة معها بشكل دائم. نقترح في هذا المشروع أن توفر برمجيات مثل Matlab يمكن من تطوير طرق الاستنتاج للسرعات و المكثفات المطلوبة لعملية الإثارة الذاتية.

Contents

Chapter One

Introduction

1.1 General:

It is well known that given a source of reactive KVA, a conventional three phase induction motor can act as a generator at a frequency which is closely related to prime-mover speed and a voltage which is determined by the amount of connected KVars. Although this fact was realized back in thirties, it is only during the last few decades that the subject of the induction generator has received considerable attention. The reason is the growing emphasis on alternative energy sources, and the need for a suitable electric generator to match the requirement of operational simplicity and cost effectiveness.

Although induction generator has already emerged as an alternative choice to the synchronous generator in such areas as stand-alone wind energy farms and remote hydro-electric applications research efforts are still directed to understanding of self-excitation boundaries of the stand-alone unit. It is generally believed that better insight into the generation requirements of a given machine will afford the cost optimization target. The work to be presented in this thesis is concerned with the self excitation boundaries of a given machine.

1.2 The Grid-Connected Induction Generator:

If a conventional 3-phase induction motor which is connected to 3-phase supply is driven by a prime-mover at speeds above that of the supply frequency, it will act as a generator feeding power back into the mains. This electro-mechanical energy reversibility can readily be seen from the conventional equivalent circuit diagram in Figure (1.1), in which the slip variable (s) will determine the mode of operation as shown by the torque-slip curve in Figure (1.2). Since the slip changes sign, the induced potential in the rotor winding will also change sign, and so will the current in the rotor winding. If we keep the idea of power flow reversal in mind, inspection of Figure (1.1) suggests that while some of the generated power is lost in the rotor winding, the remaining generated power is delivered to the stator after core losses are accounted for and the balance is delivered to supply mains in the form of alternating current going out of the stator winding.

The attractive feature of a grid-connected installation is simplicity of operation. Placing induction generator on line with an infinite-busbar involves no more than bringing the machine to speed and closing the circuit-breaker. All of the features normally used with synchronous generator for speed control, a separate source of excitation and synchronization equipments are omitted.

A grid-connected induction generator is however not without operational problems. Firstly, dynamic impacts in terms of voltage profile and stability are to be expected when the injection of the utility grid with a fluctuating and erratically-variable power source, such as that of wind energy is to be considered. Secondly, the fact that the squirrel cage machine has no field winding means that necessary magnetizing current must be supplied by the system to which it is connected. In other words the system must be capable of supplying the lagging Vars. required to establish the air-gap flux in the induction generator. This gives rise to poor operational power factor and the need for over-excited synchronous capacity, or shunt capacitors connected across the machine. The later option might lead to self-excitation problems upon disconnection and reconnection of the supply.

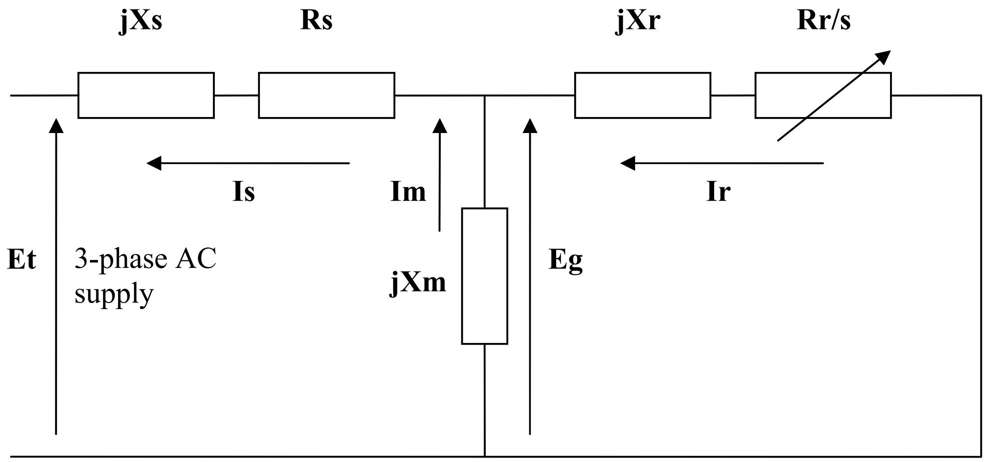

Figure (1.1)

Per-phase equivalent circuit diagram of induction machine

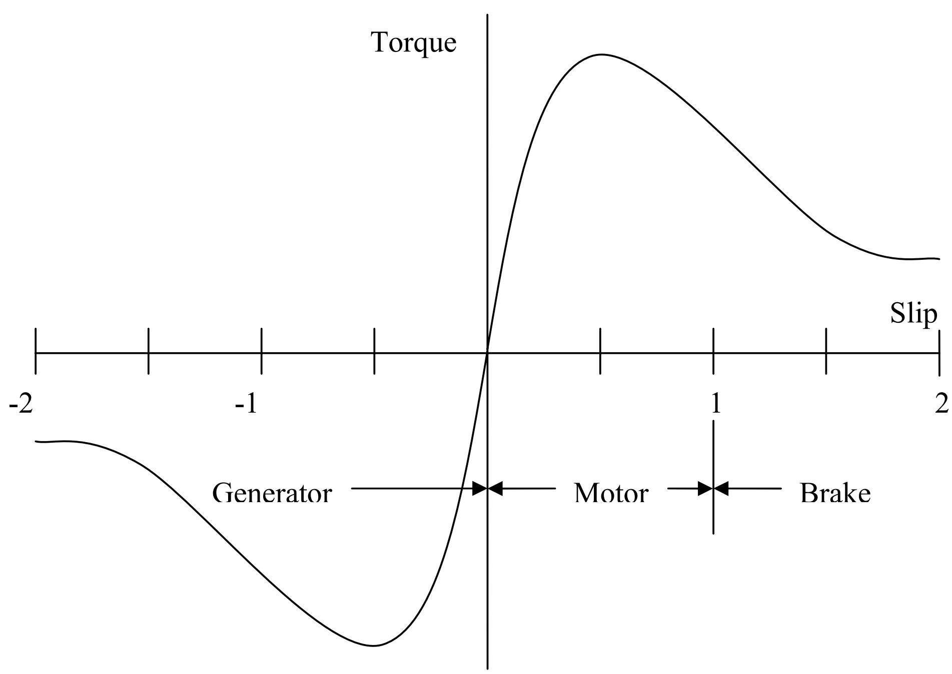

Figure (1.2)

3-phase Induction Machine Torque/Slip Curve

1.3 The Self-Excited Induction Generator:

The principle of self-excitation, by which an induction generator can be excited by means of static capacitors connected across the machine terminals, has long been known. The actual physical process by which voltage appears at the machine terminals using this method is not easy to visualize. However in order for the generation process to occur when starting up from rest, a residual magnetism must be present in the rotor. This will induce a small e.m.f. in the stator windings at a frequency proportional to rotor speed. A leading current flows in the capacitor, the same current passing the stator winding and producing an armature reaction flux assisting the original residual flux. If the capacitor is of sufficient value, the voltage continues to build-up until it is limited by saturation of the magnetic circuit of the machine. The system is therefore governed by the residual magnetism of the machine, the terminal capacitance and the speed of primemover. Once excited, the frequency of the generated voltage is in general strongly influenced by the prime-mover speed, whereas the terminal voltage is closely related to the amount of capacitance and connected load.

A simplified representation of the self-excited conditions is given in Figure (1.3) where the point of the intersection (A) of machine magnetization curve and capacitor load line occurs at the saturated region of machine characteristics. Any increase of capacitance or rotor speed reduces the slope of the capacitor load line, thus increases the generated voltage.

When supplying a load while operating with negative slip, the frequency of the stator e.m.f. is

$$f = np / (1+s) \qquad \text{----------------------} \ (1.1)$$

where n is the rotor speed (r/s) and p is the number of pole pairs. Referring to the approximation circuit in Figure(1.4) with the generator feeding into a resistive load R_l and ignoring losses, the load and generated powers are equal so that $V_2/ R_l = V_2/ (R_r/s)$. This gives the following expression for the generated frequency:

$$f = np / (1+R_r/R_l) \text{ ----------------------- } (1.2)$$

This indicates that as the load increases the frequency falls. With reference to Figure (1.5) the new generated voltage following a reduction in the frequency is at point (B). The decrease in terminal voltage will continue with increased load until de-excitation occurs.

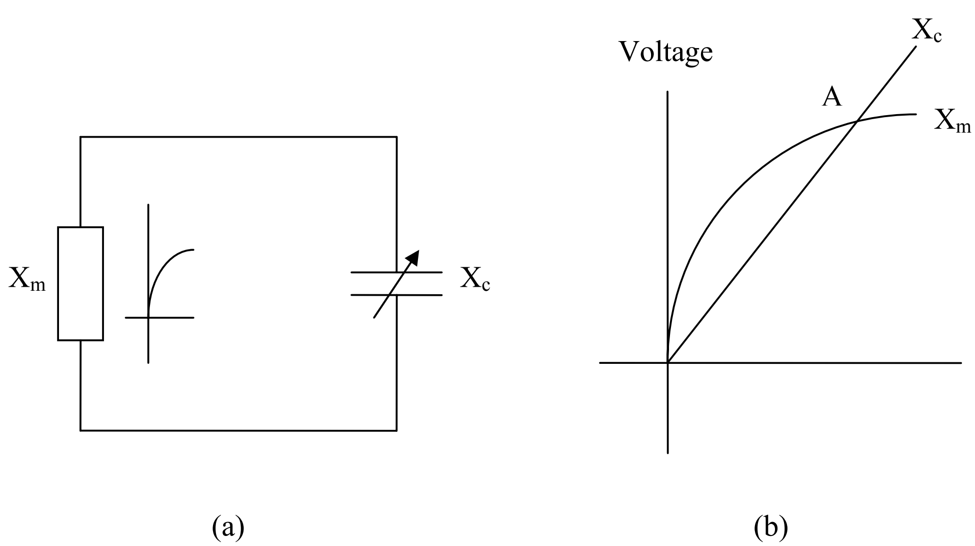

Figure (1.3)

Simplified induction generator representation and voltage equilibrium state

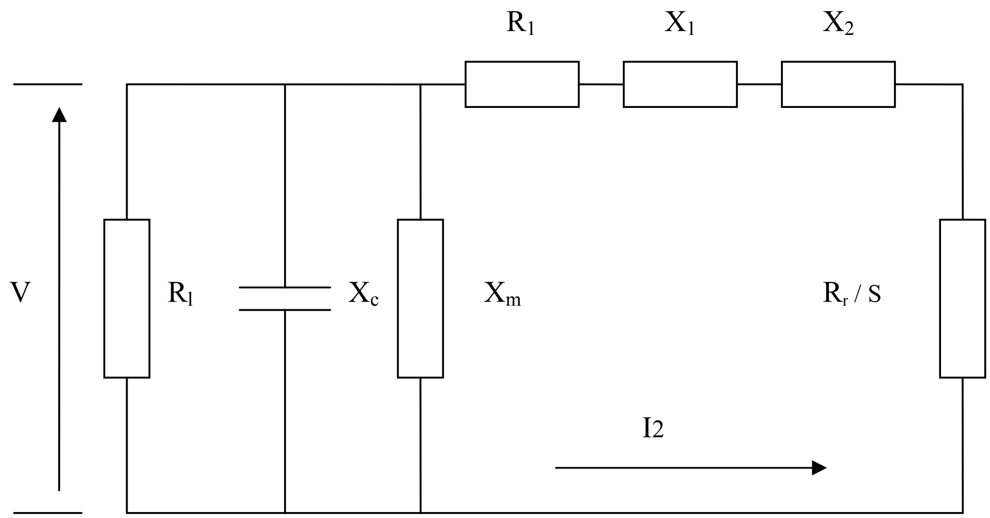

Figure (1.4)

Self-excited I.G. feeding into a resistive load

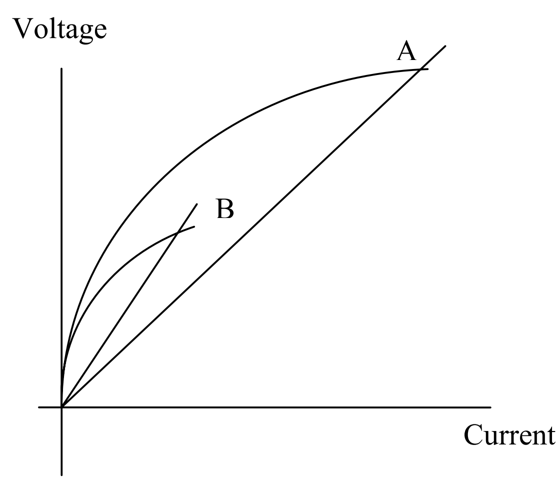

Figure (1.5)

Variation of Characteristic under Load Condition

Although the attractive features of a strong rotor construction, robustness, absence of a D.C. source of excitation, etc, might seem ideal for varying and high speed applications; it is the inability to control the output voltage and frequency that has hindered the development of isolated induction generator units. With recent advances in the field of power electronics, a number of different schemes have emerged, but these are too complicated for medium-scale wind and turbine remote hydro power applications.

1.4 Aims of the Research Project:

This project aims at conducting a number of experiments on an existing test machine with the view of observing closely the behaviour during the process of self-excitation in terms of shaft speed and terminal voltage while the shaft is being spun up from rest. The second part of the project is concerned with trying to solve system equations as exactly as possible so as to identify roots that may correspond to initial self-excitation conditions of voltage and speed.

A review of the solution methods to the self-excited machine polynomial equations is presented in Chapter Two. This is followed by the results of a set of experiments conducted in the laboratory and some of the important observations are shown in Chapter Three. The results of the attempted numerical solution to system equations are then presented in Chapter Four. The conclusion and proposed future work in this area are given in Chapter Five.

Chapter Two

Solution Models for the Capacitor – Excited Induction Generator

Models of the C-excited induction generator have been developed aiming at the minimum capacitance requirement for self-excitation and speeds at which excitation will occur for known terminal capacitance. These models are based on either the per-phase equivalent circuit that gives steady-state solutions or d-q model for the complete transient and steady-state solutions.

2.1 Capacitance Requirement for Isolated Self-Excited Induction Generator:

Advanced knowledge of the minimum capacitor value required for self-excitation of an induction generator is of practical interest. To find this capacitor value two nonlinear equations have to be solved. Different numerical methods for solving these equations are known from previous literature. However, these solutions involve some guessing in a trial-and-error procedure. A new simple and direct method is developed to find the capacitance requirement under R_L load as suggested in reference [1]. Exact values are derived for the minimum capacitance required for self-excitation and the output frequencies under no-load, inductive and resistive loads. These calculated values can be used to predict theoretically the minimum value of the terminal capacitance required for self-excitation. For stable operation C must be chosen to be slightly greater than C_{min}. Furthermore, it is found that there is a speed threshold, below which no excitation is possible no matter what the capacitor value. This threshold is called the cut-off speed. Expressions for this speed under no-load and inductive load are also given

2.2 Three-phase induction generator model:

2.2.1 Steady-state circuit:

The per-phase steady-state circuit of a self-excited induction generator under RL load is shown in Figure (2.1).

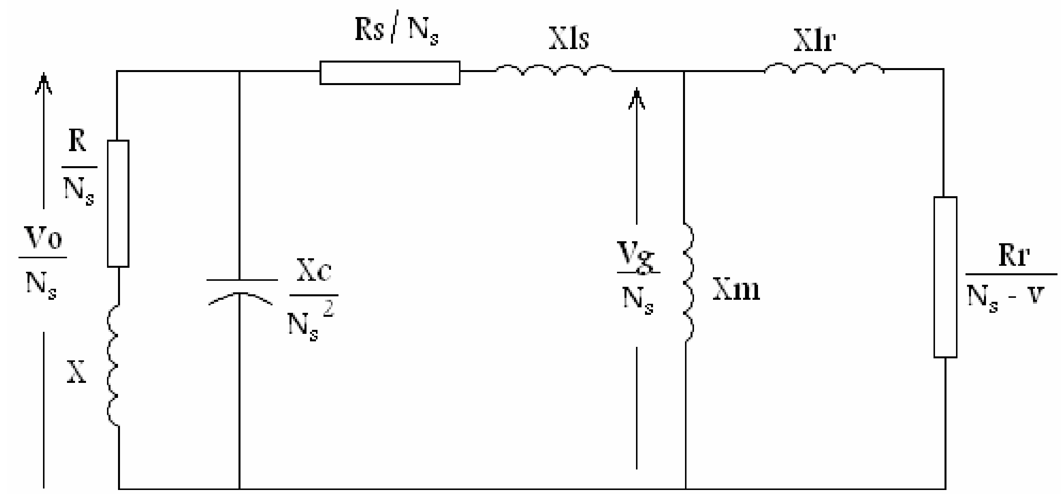

Figure (2.1)

P.u. per phase steady-state equivalent circuit of self-excited induction generator with RL load

Where; the machine core losses have been ignored. In fact, for minimum capacitance requirement, the machine must operate at the threshold of saturation. Therefore, ignoring such losses will result in no series errors in estimating $C_{min.}$ All the circuit parameters, except X_m are assumed to be constant and unaffected by saturation.

2.2.2 Mathematical model*:

For the circuit shown in Figure (2.1), the loop equation for the current I can be written as

$$IZ = 0 \qquad \text{------------------ (2.1)}$$

where, Z is the net loop impedance given by

$$Z = [((R_r/ (N_s\text{-}v) + j\ X_{lr}) \ //\ j\ Xm] + (R_s/N_s) + j\ X_{ls}$$
$$+ [(-j\ X_c/N_s^2) \ //\ ((R/N_s) + j\ X)] \qquad \text{------------- (2.2)}$$

since, under steady-state excitation I ≠ 0, it follows from equation (2.1) that Z = 0 or both real and imaginary parts of Z are zeros.

The real part yields:

$$-a_1f^3 + a_2f^2 + (a_3X_c + a_4)\ f - a_5X_c = 0 \qquad \text{----------------- (2.3)}$$

and the imaginary part yields:

$$-b_1f^4 + b_2f^3 + (b_3X_c + b_4)\ f^2 - (b_5X_c + b_6)\ f - X_cb_7 = 0 \qquad \text{---------------- (2.4)}$$

where, a_i, i = 1, 2,…, 5 and b_i, i = 1, 2,…, 7 are positive real constants.

2.2.3 Statement of the problem:

For an induction generator to be self-excited the machine has to operate in the saturation region. Therefore, for given speed and load, the terminal capacitor should have a value that X_m always lies in the saturated region. The magnetizing reactance X_m decreases with increasing saturation. Let X_{smax} be the maximum saturated reactance of the machine, which can be experimentally determined. Even though the X_m of the machine varies considerably with operating conditions (owing to saturation), the assumption

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* Some symbols in the reference paper had been changed here to avoid confusion.

of a single value X_{smax} in the analysis is acceptable, because the minimum feasible capacitor value is inevitably associated with the lowest magnetizing KVAs and magnetizing current which in turn correspond to the maximum X_m value. In practice, this Xm value corresponds to an operating point very slightly above the linear part of the machine magnetizing curve (where a stable operating point is just feasible), and hence X_{smax} will be very slightly less than the machine unsaturated magnetizing reactance.

The problem now is, given machine parameter values, speed and $X_m=X_{smax}$, to find the minimum capacitor value required for self-excitation and the frequency F that simultaneously satisfy equations (2.3) and (2.4).

To solve this problem the following method has been given:

1. For certain speed, assume a value of the terminal capacitance C and solve equations (2.3) and (2.4) for X_m and f. the initial value of C should be large enough to cause self-excitation, i.e. X_m has a value which lies in the saturated region.
2. Gradually decrease the value of C in steps, and compute X_m corresponding to each value of C. a plot of X_m against C is thus derived.
3. Cmin is obtained from such a plot as the intersection of the X_m against C curve and the line $X_m = X_{min}$.

2.2.4 Proposed method to find general solution for C_{min}:

In this section a direct method to find C under general R_L load is developed. This new approach is different from the iterative discussed in reference. This new approach is used later to find exact expressions for C_{min} under no-load, inductive and resistive loads. Let $X_m = X_{smax}$. Equation (2.3) can be rewritten as

$$X_c = [A_1f^3 - A_2f^2 - A_4f] / [A_3f - A_5] \qquad \text{----------------- (2.5)}$$

similarly equation (2.4) can be written as

$$X_c = [B_1f^4 - B_2f^3 - B_4f^2 + B_6f] / [B_3f^2 - B_5f - B_7] \text{ --------- } (2.6)$$

where all the coefficients are evaluated at $X_m = X_{smax}$.

Since we are looking for C [$=1/ (2\Pi f_b Z_b X_c)$] that simultaneously satisfies equations (2.5) and (2.6) it follows that:

$$[A_1f^3 - A_2f^2 - A_4f] / [A_3f - A_5] = [B_1f^4 - B_2f^3 - B_4f^2 + B_6f] / [B_3f^2 - B_5f - B_7]$$

this had been reduced to

$$A_4f^4 - \alpha_3f^3 + \alpha_2f^2 - \alpha_1f + \alpha_0 = 0 \qquad \text{----------------------- } (2.7)$$

Let $\{f_i, i \leq 4\}$ be the set of positive real roots of equation (2.7), and let $\{C_i, i \leq 4\}$ be the corresponding set of positive capacitor values (by substituting the roots of equation (2.7) in equation (2.5) or (2.6)). Since all these values of C (at a given values of v and X_{msat}) are sufficient to guarantee self-excitation of the induction generator, it follows that the minimum capacitor value required is given by:

$$C_{min} = \min \{C_i, i \leq 4\}$$

If equation (2.7) has no real roots, then no excitation is possible in fact, as will be shown later, there is a minimum speed value, below which equation (2.7) has no real roots. Correspondingly, no excitation is possible.

To summarize the result; given the machine parameters, X_{msat} and the speed v, solve equation (2.7) to find the per unit frequency F which may have more than one value. For each value of F find the corresponding capacitor value, and then take the minimum of these capacitor values. This value is C_{min}. Consider the following example:

Machine data are

$R_s = 0.071$ p.u.

$R_r = 0.0881$ p.u.

$X_{ls} = X_{lr} = 0.1813$ p.u.

$X_{msat} = 3.23$ p.u.

$Z_b = 43.3\ \Omega$

$N = 1800$ rev/min

$f_b = 60$ Hz

load is

$R = 1.0$ p.u.

$X = 2.0$ p.u.

and speed is

$V = 1.0$ p.u.

equation (2.7) is reduced to

$$8.4312\ F^4 - 13.1859\ F^3 + 6.7819\ F^2 - 2.8192\ F + 0.8855 = 0$$

solving this equation gives the two real roots

$F_1 = 0.5292$ p.u. (31.7492 Hz)

and

$F_2 = 0.9795$ p.u. (58.768 Hz)

substituting in equation (2.5) or (2.6) yields

$C_1 = 657\ \mu F$ and $C_2 = 45.698\ \mu F$

or

$C_{min} = 45.698\ \mu F$

at a frequency of 58.768 Hz.

The degree of equation (2.7) determines the number of capacitor values from which the minimum value is selected. In the above example, the polynomial is of degree 4. Therefore, four real roots are possible. These roots, if they exist, are positive. In fact, equation (2.7) can be rearranged as

$$A_4F^4 + \alpha_2F^2 + \alpha_0 = \alpha_3F^3 + \alpha_1F \qquad \text{----------------- (2.8)}$$

which can be viewed as the intersection of two polynomials, with the general shape as illustrated in Figure (2.2).

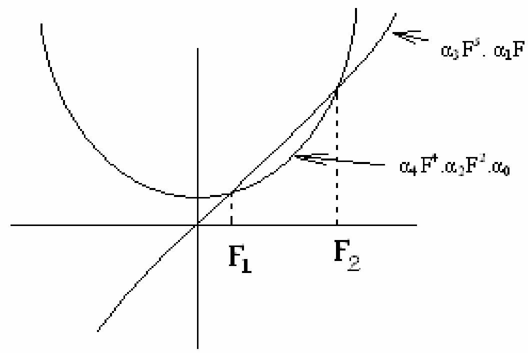


Figure (2.2)

Illustration Sketch of Equation (2.8)

These two curves, if they meet, intersect in the range $F_1 \leq F \leq F_2$. In fact, as will be shown later, at no-load and inductive load there are only two real roots.

However, for the general R_L case all numerical result considered indicate that only two real roots exist. This restricts the set to two capacitor values. An analytical solution of equation (2.7) exists. However, numerical techniques may be more convenient to use. The two curve of equation (2.8) do not necessarily intersect. In fact, the coefficients α_i, i = 0, 1,…, 4 of equation (2.8) in addition to the machine parameters, are functions of the p.u. speed v and the load. Decreasing the speed results in a situation where the two polynomials of equation (2.8) do not intersect, resulting four imaginary roots of equation (2.7). The value of this speed will be called the cutoff speed v_c.

All roots of equation (2.7) if they exist, are positive and are bounded by v. if F were to be greater than v, then from Figure (2.1) R_r / (F-v) is strictly positive, and therefore no excitation is possible. To summarize; if v ≥

vc, the roots of equation (2.7) are possible and bounded by v. this result implies that there are the most four capacitor values to select from, and the problem now is restricted to choosing the minimum of these values. In the special cases, no-load, inductive and resistive loads, it is possible, as discussed below, to show that X_c is an increasing function of F (or C is a decreasing function of F). Therefore, choosing $F_{max} = \max [F_i, i \leq 4]$ will result in the minimum capacitor value required for self-excitation of the induction generator.

To simplify the analysis which is follows it is assumed that $X_{ls} = X_{lr}$. This assumption is normally valid in induction-machine analysis. Next the special cases of no-load, inductive and resistive loads are considered.

2.2.5 Special cases:

2.2.5.1 No-load capacitance requirement:

This case can be derived from the general case by letting X = 0 and taking the limit as R goes to infinity. In this case, equations (2.5), (2.6) and (2.7) are reduced to

$$X_c = AF^2 - B\,(F/(F-v)) \quad \text{------------------------------------} \quad (2.9)$$

$$X_c = v + DF\,(F-v) \quad \text{-----------------------------------} \quad (2.10)$$

and

$$F^2 - (1 + D / (E + D - A)\, v\, F + (B + V^2 D) / (E + D - A) = 0 \quad \text{---} \quad (2.11)$$

where

$A = X_{ls} + X_{smax} // X_{lr}$

$B = R_s R_r / (X_{lr} + X_{smax})$

$D = R_s (X_{lr} + X_{smax}) / R_r$

$E = X_{ls} + X_{smax}$

equation (2.11) can be solved to yield

$$F = (v/2)\ \{1 + [(R_s/R_r)\,(1 + (X_{lr}/X_{smax}))^2 \pm \sqrt{(1 - (v_c/v)^2)}]\ /$$

$$[1 + (R_s/R_r)\ (1 + (X_{lr} / X_{smax}))^2]\} \quad \text{-----------} \ (2.12)$$

where

$$v_c = 2R_s/X_{smax}\ [\sqrt{\ } (R_r/R_s + (1 + (X_{lr} / X_{smax}))^2)]$$

obviously both roots are real and positive, provided that $v \geq v_c$.

This speed v_c, below which the machine will not operate, is the cutoff speed. For the previous machine with no-load, v_c = 0.0675 p.u. (or 121.47 rev/min).

To show that X_c is an increasing function of F dX_c/dF must be positive. But

$$dX_c/\ dF = 2AF + (Bv) / (F\text{-}v)^2$$

which is always positive for any $0 < F < v$, implying that X_c is an increasing function of F. since minimizing C corresponds to minimizing X_c, it follows that Fmax = max (F1, F2) minimizes C. therefore, from equation (2.10)

$$C_{min} = [2\ \Pi\ f_b\ Z_b\ (X_{lr} + X_{smax}) * ((R_s/R_r)\ F_{max}\ (F_{max} - v) + F^2{}_{max})]^{-1} \text{ -- } (2.13)$$

where

$$F_{max} = (v/2)\ \{1 + [(R_s/R_r)\ (1 + (X_{lr}/X_{smax}))^2 + \sqrt{\ } (1 - (v_c/v)^2)] /$$

$$[1 + (R_s/R_r)\ (1 + (X_{lr} / X_{smax}))^2]\} \quad \text{-----------} \ (2.14)$$

In reference [3], the machine of previous example was tested, and the experimental results are plotted in Figure (2.3), together with the calculated results.

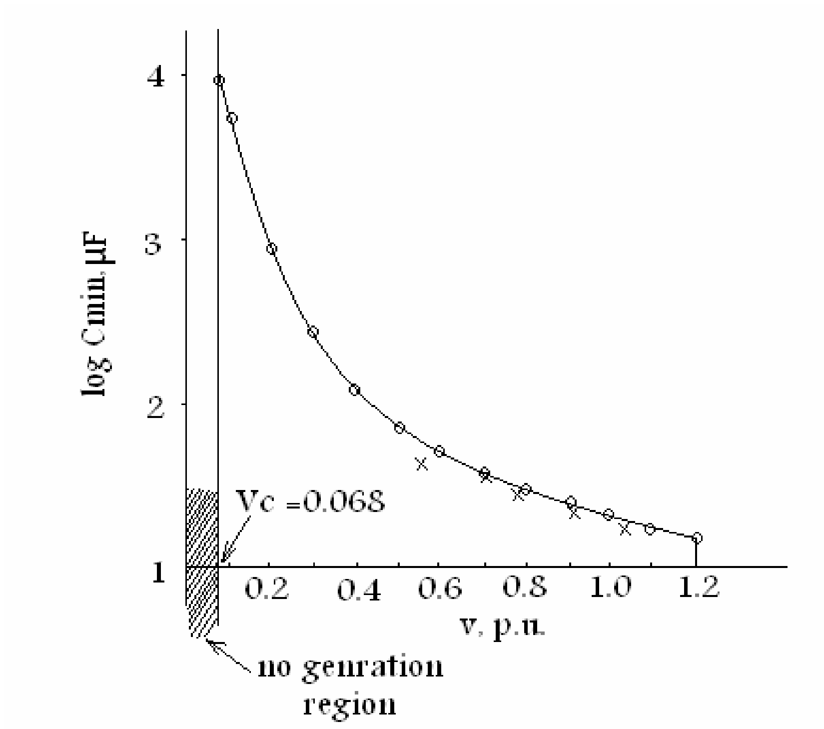


Figure (2.3)

Speed against minimum capacitance

________ Calculated   x x x Experimental

together with the calculated results. In this figure, the region of no generation is shown. Although the capacitor value at v_c is very large for practical verification, it is usually desirable to know the limits on the machine performance.

If, however, $v >> v_c$, then equation (2.14) can be expressed by

$$F = v - \varepsilon$$

where $\varepsilon > 0$ is small number which depends on (v_c / v) or

$$(R_r / (N_s - v)) = (R_r / \varepsilon)$$

is very large. By finding the equivalent series impedance of $\{j\ X_{msat} // [R_r / (N_s - v)]\}$ and neglecting all terms containing $(N_s - v)^2$, the circuit of Figure (2.1) is reduced to the circuit shown in Figure (2.4).

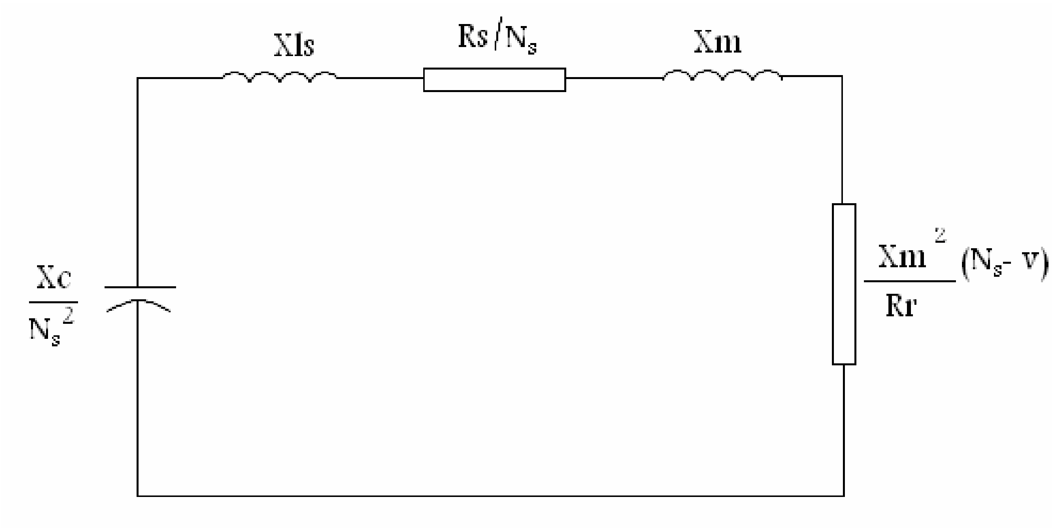


Figure (2.4)

Approximate equivalent Circuit of Figure (2.1) at no-load

From this figure and by letting the real and imaginary parts of the loop impedance be zero, it follows that:

$$F^2 - v F + (R_s R_r) / X^2_{smax} = 0 \quad \text{-----------------------} \ (2.15)$$

and

$$X_c = F^2 (X_{lr} + X_{smax}) \quad \text{---------------------------------} \ (2.16)$$

solving equation (2.15) gives

$$F = ½ \{v \pm \sqrt{[v^2 - (4 R_s R_r / X^2_{smax})]}\}$$

this gives approximate expression for the cutoff speed

$$v_c \approx \sqrt{[4 R_s R_r / X^2_{smax}]}$$

and

$$C_{min} \simeq [2\Pi f_b Z_b (X_{smax} + X_{lr}) F^2_{max}]\text{-}1$$

where

$$F_{max} = ½ \{v + \sqrt{[v^2 - (4 R_s R_r / X^2_{smax})]}\}$$

A similar approximation was used in reference [3] bases on some experimental observations. Both approximations, here and in reference 3, are valid only if the condition of approximation is satisfied, i.e. the speed must be much greater than the cutoff speed.

2.2.5.2 Inductive load capacitance requirement:

A closed formula for C_{min} in this case is possible. This case can be derived from the general case by letting R = 0. In this case equations (5), (6) and (7) can be rewritten as

$$F^2 - v\,F\,(1 + \alpha) + \alpha\, v^2 + ß = 0 \qquad \text{--------------} \ (2.17)$$

$$X_c = \frac{X\,[X//\,(X_{lr} + X_{lr}//X_{smax})]\,F^2 * [F^2 - v\,F + (R_r\,R_s) / (X_{smax} + X_{lr})\,(X_{lr} + X_{lr}\,//\,X_{smax})]}{[F^2 - v\,F + (R_r\,R_s) / (X_{smax} + X_{lr})\,(X + X_{lr} + X_{lr}\,//\,X_{smax})]}$$

and

$$X_c = [X\,(X_{lr} + X_{smax})\,(R_r + R_s) / R_r\,(X_{lr} + X_{smax} + X) + R_s\,(X_{lr} + X_{smax})]$$
$$* [F - (R_s\,v / (R_r + R_s))]$$
$$/ [F - [R_s\,(X_{lr} + X_{msat})\,v] / [R_r\,(X_{lr} + X_{smax} + X) + R_s\,(X_{ls} + X_{smax})]] \qquad \text{--------------} \ (2.18)$$

where

$$\alpha = [1 + (R_r / R_s)\,(X_{smax} / (X_{smax} + X_{lr}))^2]^{-1}$$
$$ß = [(R_s\,R^2{}_r) / (R_r\,X^2{}_{smax} + R_s\,(X_{smax} + X_{lr})^2)]$$

equation (2.17) yields two positive real roots F_1 and F_2 with $0 < F_1 \leq F_2 < v$, provided that $X > 0$ and $v \geq v_c$, with the cutoff speed being given by

$$v_c = (2\,R_s / X_{smax}) * \sqrt{\,}\,[(R_r / R_s) + (1 + (X_{lr} / X_{smax}))^2]$$

this turns out to be the same as the cutoff speed for the no-load case. It can be shown that X_c is an increasing function of F for $0 < F < v$. Correspondingly,

$$C_{min} = [(2 \Pi f_b Z_b X) * [1 + ((R_r X) / (R_r (X_{ls} + X_{smax}) + R_s (X_{lr} + X_{smax})))]^{-1} * F^2_{max} * [(F_{max} – v (1 + (R_r / R_s))^{-1}) / (F_{max} – v [1 + (R_r (X_{smax} + X_{lr} + X)) / (R_s (X_{smax} + X_{lr}))]^{-1})]]^{-1} \quad \text{-------- (2.19)}$$

where

$$F_{max} = (v/2) \{1 + [(R_s/R_r) (1 + (X_{lr}/X_{smax}))^2 + \sqrt{} (1 – (v_c/v)^2)] / [1 + (R_s/R_r) (1 + (X_{lr} / X_{smax}))^2]\}$$

by taking the limit as $X \rightarrow \infty$, C_{min} (equation (2.19)) for the inductive load approaches the C_{min} of the no-load case (equation (2.13)).

If $v >> v_c$, then F is closed to v and a similar approximation to the no-load case can be used here, leading to

$$v_c \simeq \sqrt{} [(4 R_s R_r) / (X^2_{smax})]$$

$$F_{max} = ½ \{v + \sqrt{} (v2 – (4 R_s R_r) / (X^2_{smax}))\}$$

and

$$C_{min} \approx 4 [2 \Pi f_b Z_b (X // (X_{lr} + X_{smax})) * (v + \sqrt{} (v2 – (4 R_s R_r) / (X^2_{smax})))]^{-1} \quad \text{--------- (2.20)}$$

<u>2.2.5.3 Resistive load capacitance requirement:</u>

This can be derived from general case by letting X = 0. In doing so equations (2.5) and (2.6) are reduced to

$$X_c = K_1 F [(F^2 – v F – K_2) / (F – v K_3)] \quad \text{--------------------- (2.21)}$$

$$X_c = -J_1 F [(F – v J_2) / (F^2 – v F – J_3)] \quad \text{-------------------- (2.22)}$$

where

$$K_1 = R (X_{ls} + X_{smax} // X_{lr}) / (R_r + R + R_s)$$

$$K_2 = R_s R_r / [(X_{smax} + X_{lr}) (X_{ls} + X_{smax} // X_{lr})]$$

$K_3 = (R + R_s) / (R_r + R + R_s)$

$J_1 = R\,(R_r + R_s) / (X_{ls} + X_{smax} // X_{lr})$

$J_2 = R_s / (R_r + R_s)$

$J_3 = R\,(R_r + R_s) / (X_{smax} + X_{lr})\,(X_{ls} + X_{smax} // X_{lr})$

To find C_{min}, X_c is eliminated from equations (2.21) and (2.22), yielding an equation of F only. Solving this equation gives the p.u. frequency. Substituting the solution in equations (2.21) or (2.22) gives the required value of C.

In the following it is shown that $F_{max} = \max [F_i,\ i \le 4]$ will give the minimum value of C_{min}. To this end it is sufficient to show that either equation (2.21) or (2.22) is an increasing function in the range $F_1 \le F \le F_2$, where F_1 and F_2 are the solutions to equation (2.7) when X = 0. Equations (2.21) and (2.22) can be rewritten as

$X_c = K_1 F\,[(F - Z_2)\,(F - Z_5) / (F - P_4)]$

$X_c = -J_1 F\,[(F - Z_3) / (F - P_1)\,(F - P_6)]$

where

$P_1 = ½\,[v - \sqrt{(v^2 + 4 J_3)}]$

$P_4 = v K_3$

$P_6 = ½\,[v + \sqrt{(v^2 + 4 J_3)}]$

$Z_2 = ½\,[v - \sqrt{(v^2 + 4 K_2)}]$

$Z_3 = v J_2$

$Z_5 = ½\,[v + \sqrt{(v^2 + 4 K_2)}]$

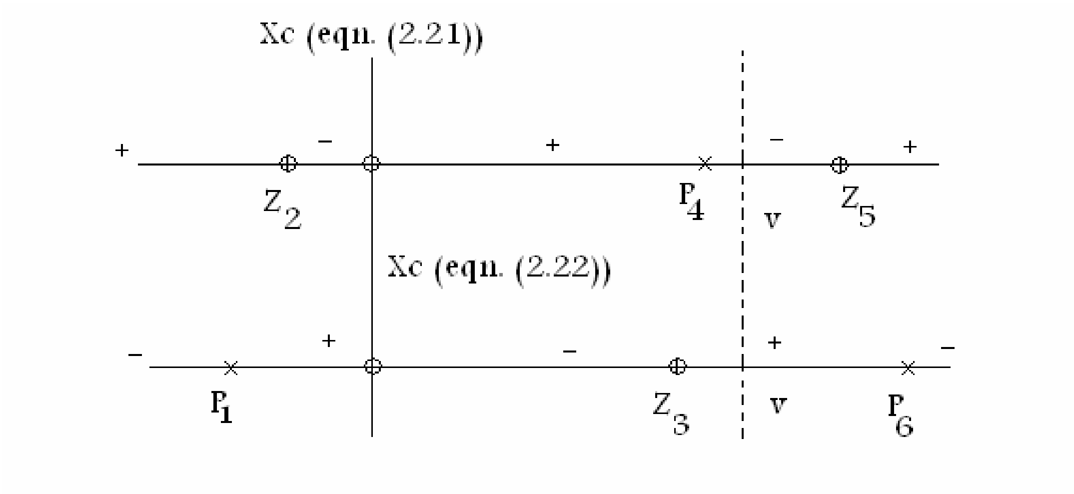


Figure (2.5)

Poles and zeros of equations (2.21) and (2.22)

the signs of X_c are indicated on the figure.

The only region where the both functions (equations (2.21) and (2.22)) have the same sign, and therefore give a possible solution is

$$Z_3 < F < P_4$$

i.e. the set of solutions of equation (2.7) lies in

$$v\,(R_s / (R_s + R_r)) < F < v((R + R_s) / (R + R_s + R_r))$$

in this range X_c (equation (2.22)) is an increasing function of F. in fact

$$X_c = -J\,F\,(F - Z_3) / [(F + P_1)\,(F - P_6)]$$

and

$$dX_c / dF = (-J / [(F + P_1)^2\,(F - P_6)^2]) * \{F\,(F + P_1)\,(F - P_6) + P_1\,(F - P_6)\,(F - Z_3) - F\,(F - Z_3)\,(F + P_2)\} > 0$$

therefore

$$C_{min} = [2 \Pi f_b Z_b K_1 F_{max} (F^2_{max} - v F_{max} - K_2) / (F_{max} - v K_3)]^{-1} \text{ --- } (2.23)$$

where F_{max} is the maximum of real roots of equation (7) evaluated at X = 0.

<u>2.2.6 Concluding Remarks about the Equivalent Circuit model:</u>

A new simple and direct method of obtaining the minimum capacitor value required for self-excitation of the induction generator under the RL load has been introduced. These values can be used to predict theoretically the minimum values of the terminal capacitance required for self-excitation. Of course, for stable operation of the machine C must be chosen slightly greater than C_{min}. Exact expressions for the capacitor values under no-load, inductive and resistive loads and corresponding output frequencies are also derived.

The theoretical results of no-load derived here show a good agreement with the experimental measurements that have been carried out previously. The results also show that the machine will operate only if the speed is greater than a threshold called the cutoff speed. Exact expressions under no-load and inductive load are given. This concept gives theoretical limits to the lowest speed at which the induction generator can be driven.

It is believed that simulation of capacitor-excited machine based on the d-q model should give better conclusions about the excitation voltages and speeds. The results can demonstrate both transient and steady-state conditions as shown in Figure (2.6) and Figure (2.7). These recordings are extracted from a paper advertising the use of a general simulation software package called SIMNON [2, 4]. The development of such software is beyond the scope of this project.

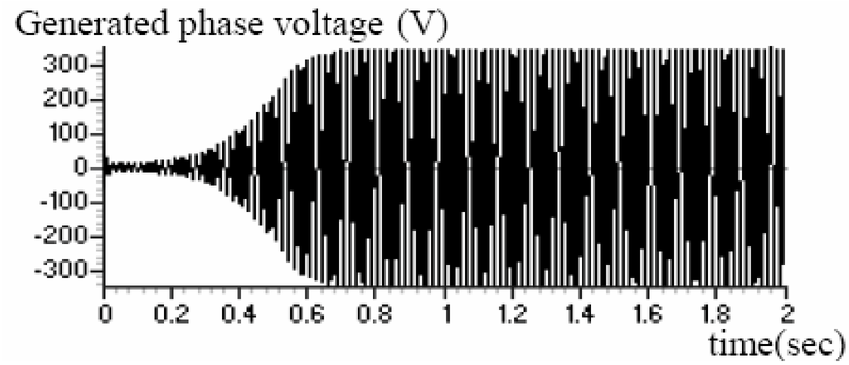


Figure (2.6)

Simulated self-excitation at 1500rpm and 60μF.

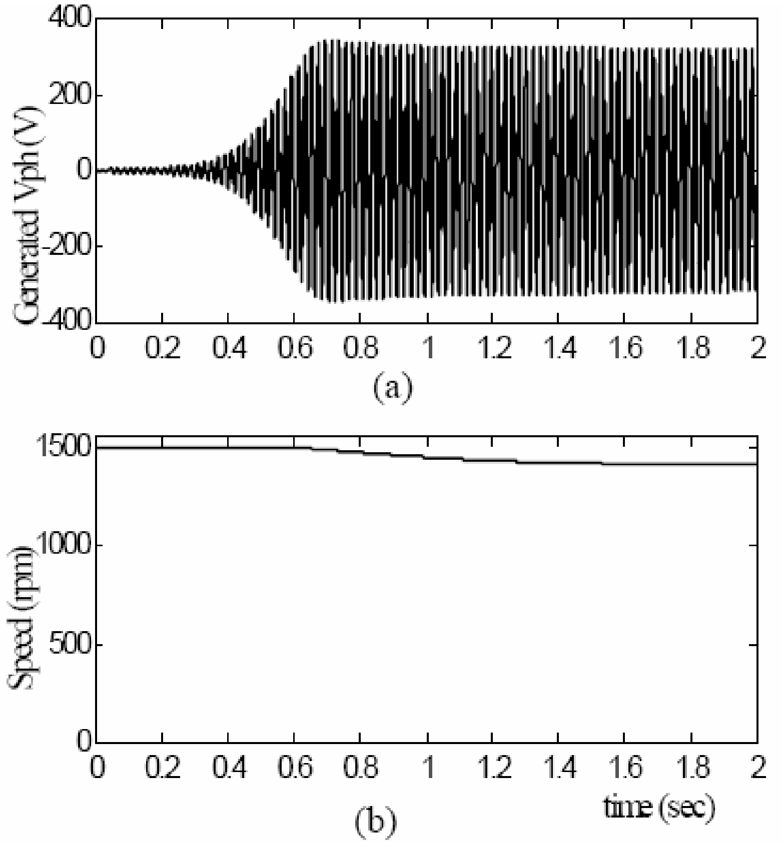


Figure (2.7)

Measured self-excitation at 60μF (a) generated voltage (b)speed.

Chapter Three

Experimental Observations on the Self-Excited Induction Generator-Primemover System

This chapter presents the results of experimental observations on the self-excitation process and boundaries of the induction generator when the machine is driven from rest while the terminal capacitors are permanently connected. Close observations for the low-rate and high-rate excitation speeds show that the rate of primemover ramp may affect the initial excitation and cut-off speeds.

Details of the experimental machine and the capacitor bank used for the experiments are given.

3.1 Details of the Experimental Machine:

The experimental set chosen for the induction generator- primemover system was a 5 KVA; 50 Hz Induction motor laboratory set coupled to a 9 h.p., 220 v D.C. motor drive. Details of the system are as given in Table (3.1). The parameters shown for the induction machine were obtained from usual no-load and locked rotor tests. Table (3.2) shows details of the 3-phase capacitor bank according to the groups and subgroups shown in Figure (3.1) and Figure (3.2).

Table (3.1)

Particulars of the Experiment Machine Set

Output Power	5 KVA
Supply	3-phase, 220 V
Poles	4
Frequency	50
X1	2.11
X2	2.11
R1	0.73
R2	1.59
X_m	65
Z_b	9.68

Table (3.2)

Capacitor Bank Values (Micro Farad)

Group	Capacitor I	Capacitor II	Capacitor III	Approximated value
A	19.8	19.9	19.8	20
B	24.1	24.1	24.1	24.1
C	24.5	24.5	24.5	24.5
D	25.0	24.7	25.0	25
E	30.6	30.3	28.8	30
F	30.8	31.0	31.4	31

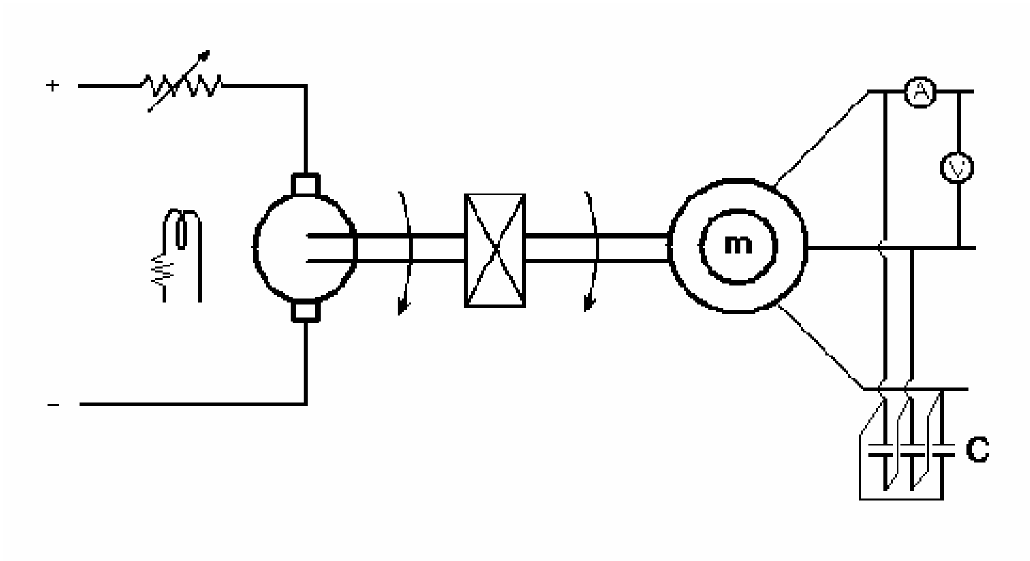


Figure (3.1)

The Induction Generator- primemover System

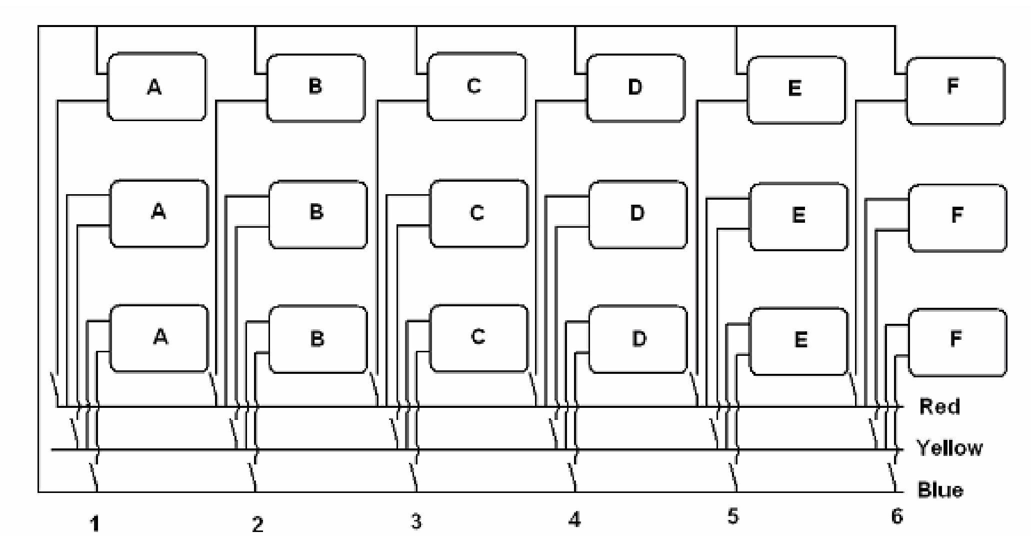


Figure (3.2)

Balanced 3-phase Capacitor Bank Schematic

3.2 Low acceleration Rate - Experiment (No.1):

In this experiment the machine was driven on low rate acceleration; which is governed with an external variable resistance of the DC motor drive. In some speed an excitation process happened and beyond which the rotating speed came down to a steady-state operational value. In this steady-state operational speed, the generator produced oscillatory voltages the average value of them was recorded.

The next step in this experiment was to slow down the rotor speed till the produced voltage value was approximately zero. The corresponding speed signifying end of generation is the cut-off speed.

This experiment was repeated with different capacitor values with the results shown in tables (3.3) and (3.4).

Table (3.3)

Capacitance Effect on Self-excitation Three Modes of Speeds (a)

C Micro Farad	First over shoot Speed	Cut – Off Speed	Steady-state Speed
20	1740	920	1220
30	1490	900	1000
45	1410	740	850
75	1180	320	650
80	1120	280	630
100	1050	220	580

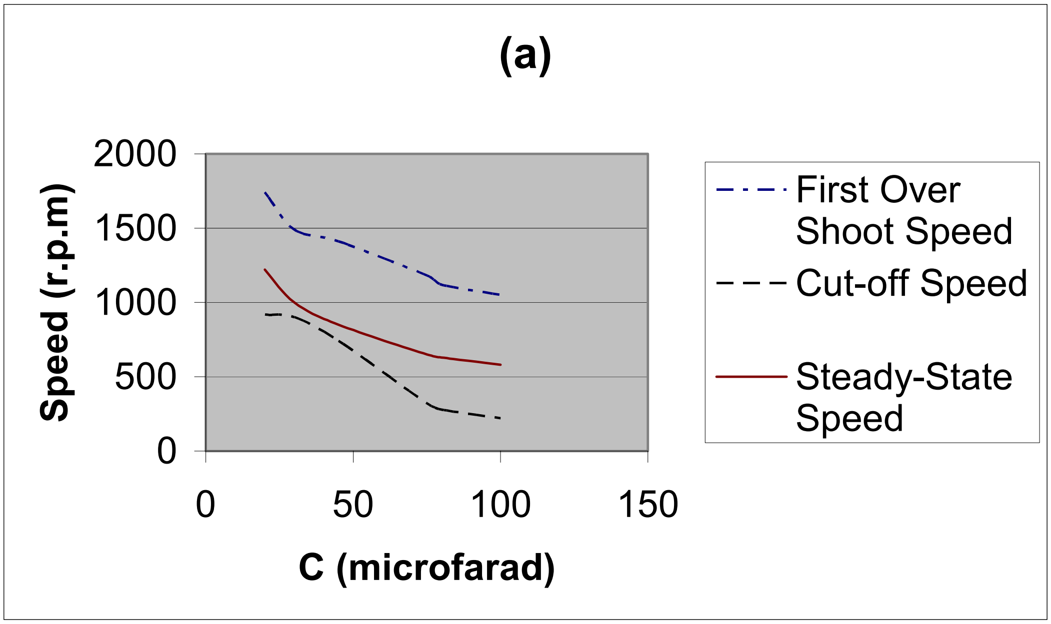


Figure (3.3)

Capacitance effect on excitation (a)

Table (3.4)

Capacitance Effect on Self-excitation Three Modes of Speeds (b)

C Micro Farad	First over shoot Speed	Cut – Off Speed	Steady-state Speed
20	1740	920	1220
30	1490	900	1000
45	1410	740	850
110	1350	180	560
123.5	1350	120	520
130	1350	100	510
154.5	1400	80	490

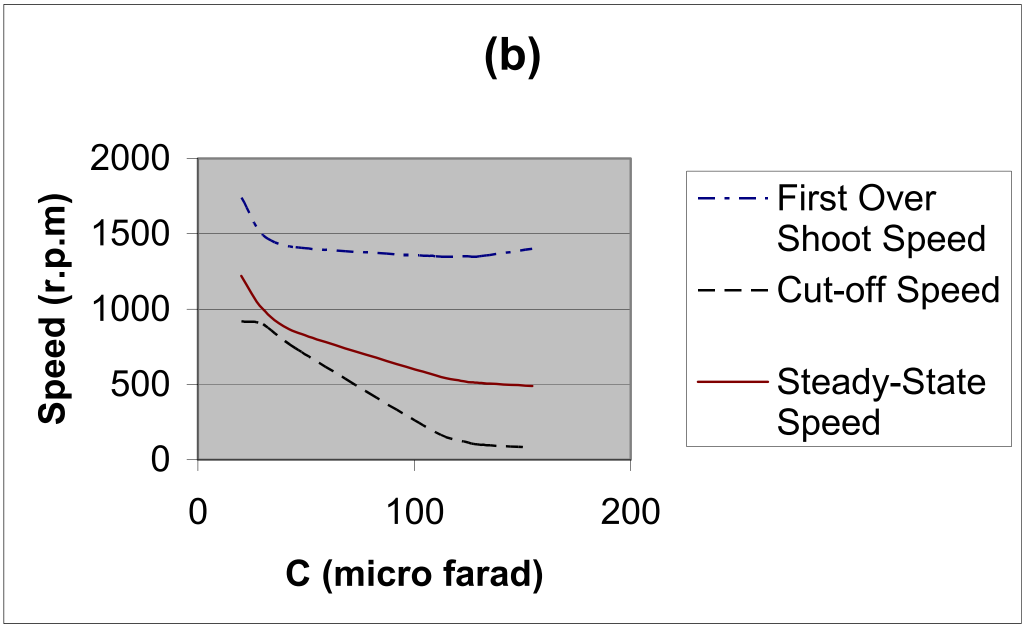


Figure (3.4)

Capacitance effect on excitation (b)

3.3 Two Trials Low Acceleration Rate - Experiment (No.2):

In this experiment a different strategy for the speed ramp rate was adopted. This consisted of driving the machine to self-excitation and cut-off as in the previous experiment. The machine was then re-accelerated with the same rate until excitation occurred. The results for the first trial and second trial are as shown in Table (3.5).

Table (3.5)

Operating Regions / Steady-state Speed & Voltage for Variable External Capacitance

C μF	Low rate 1st trial			Low rate 2nd trial		
	First over shoot Speed	Steady-State Speed / V	Cut – Off Speed	First over shoot Speed	Steady-State Speed / V	Cut – Off Speed
20	1840	1220/290	920	1900	1240/315	950
30	1450	1000/200	900	1680	1020/250	890
45	1410	850/195	740	1520	860/220	740
50	1100	800/170	700	1200	800/150	700
61	1050	720/135	450	1250	710/145	700
69	1020	680/110	320	1140	680/110	350

Notice 2^{nd} trial different readings for the low capacitor values. These differences in speed and voltage measurements disappear with the higher capacitor values. Note also shift in cut-off speeds with the 2^{nd} trial, and steady state voltage rise except for the last 69 μ.F. readings.

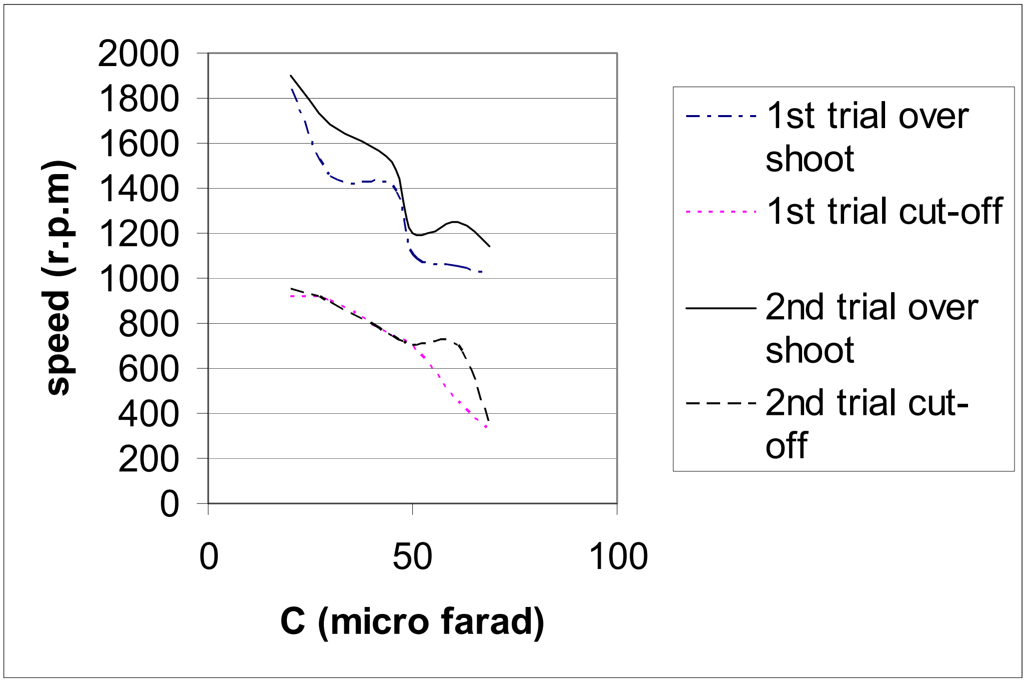


Figure (3.5)

Two Trials Low Acceleration Rate

3.4 High Acceleration Rate - Experiment (No.3):

In this experiment, the machine was powered with resistor values pre-set to those at which the low rate speeds were recorded.

In this condition, the acceleration of rotor speed was very high compared to those in experiments (1) and (2).

This process had been repeated more than once for three different capacitor values. The following table and graphs show the results.

Table (3.6)

Low and high acceleration rate tests results

C µ.F	**Low rate 1st trial**			
	First over shoot Speed	over shoot voltage pulse	Steady-State Speed	Steady-State Speed voltage
110	1350	540	560	135
123.5	1350	530	520	130
154.5	1400	540	490	132
C µ.F	**Low rate 2nd trial**			
	First over shoot Speed	over shoot voltage pulse	Steady-State Speed	Steady-State Speed voltage
110	1480	610	580	155
123.5	1500	600	540	150
154.5	1530	580	520	182
C µ.F	**Low rate 3rd trial**			
	First over shoot Speed	over shoot voltage pulse	Steady-State Speed	Steady-State Speed voltage
110	1670	680	600	185
123.5	1520	600	560	180
154.5	1520	580	600	240
C µ.F	**High rate**			
	First over shoot Speed	over shoot voltage pulse	Steady-State Speed	Steady-State Speed voltage
110	720	370	620	210
123.5	620	300	620	230
154.5	720	335	700	335

The low rate 1^{st}, 2^{nd} and 3^{rd} trials shown are given for C-values that are higher than that shown in Table (3.5). In addition, and for purposes of comparison with the high rate results, the initial overshoot voltage is recorded. Note the seemingly shift in first overshoot speed, overshoot voltage pulse, steady-state speed and steady-state voltage with the number of trials for a given C except for the overshoot in the 3^{rd} trial obtained with C = 154.5 μ.F.

For the high acceleration rate, the first overshoot speed of concern in this investigation is lower than that for the low rate case for a given C as expected. Notice the increase in steady-state voltages and speeds compared with the low rate case although the 1^{st} overshoot speed goes down by nearly a half. Note also the occurrence of zero difference between overshoot voltage and steady-state voltage. The next figures show the scalar difference in voltage (ΔV) and that in speed (Δv) for the low rate and high rate, overshoot levels in low rate trials, overshoot levels and voltage pulse in low rate and high rate, and steady-state voltages in low rate trials and high rate trial.

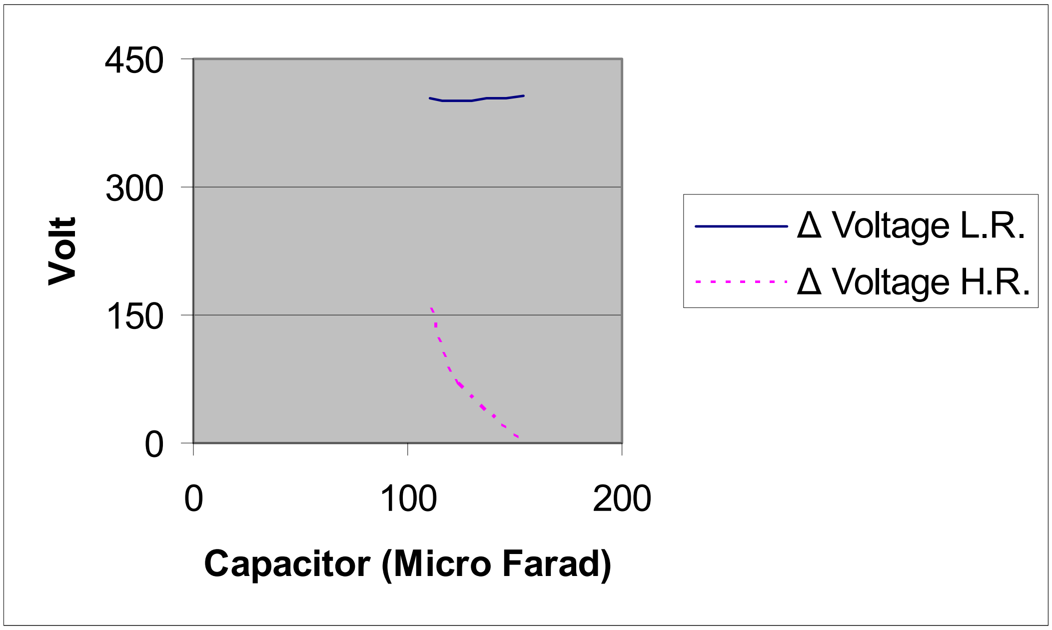


Figure (3.6)

Scalar Differences between overshoot and Steady-state voltage in Low Rate and High Rate

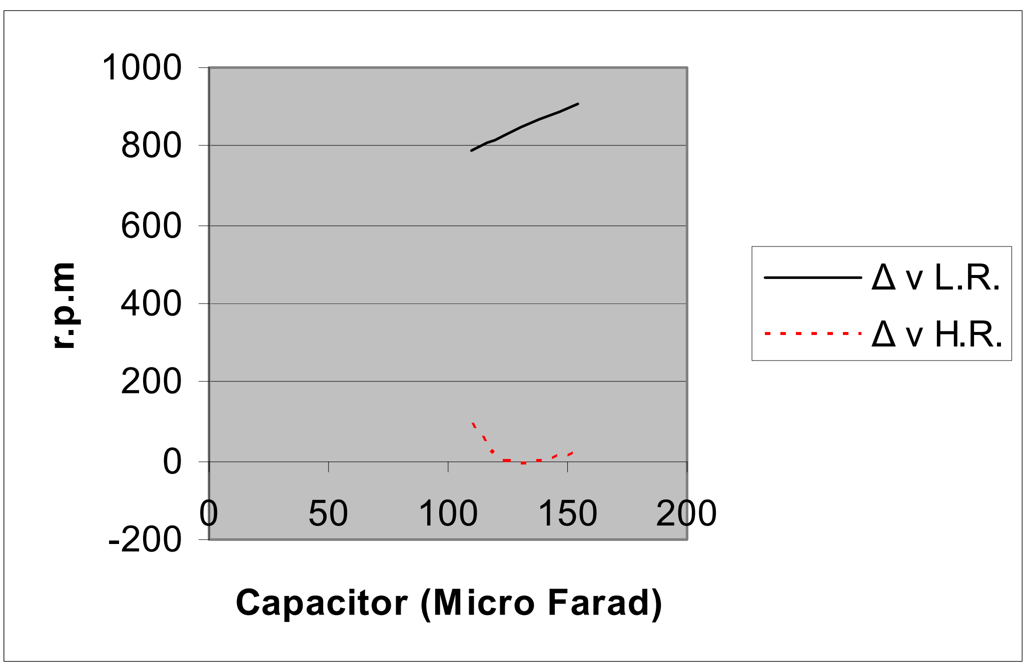


Figure (3.7)

Scalar Differences between Overshoot and Steady-state Speed in Low Rate and High Rate

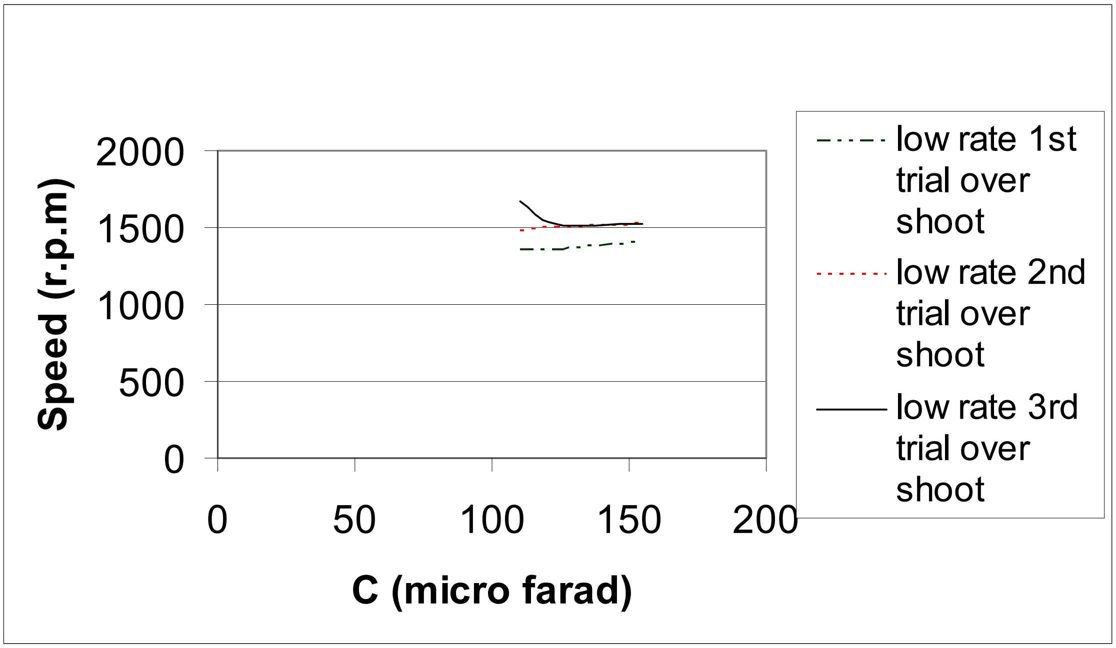


Figure (3.8)

Low Rate Three Trials Overshoot

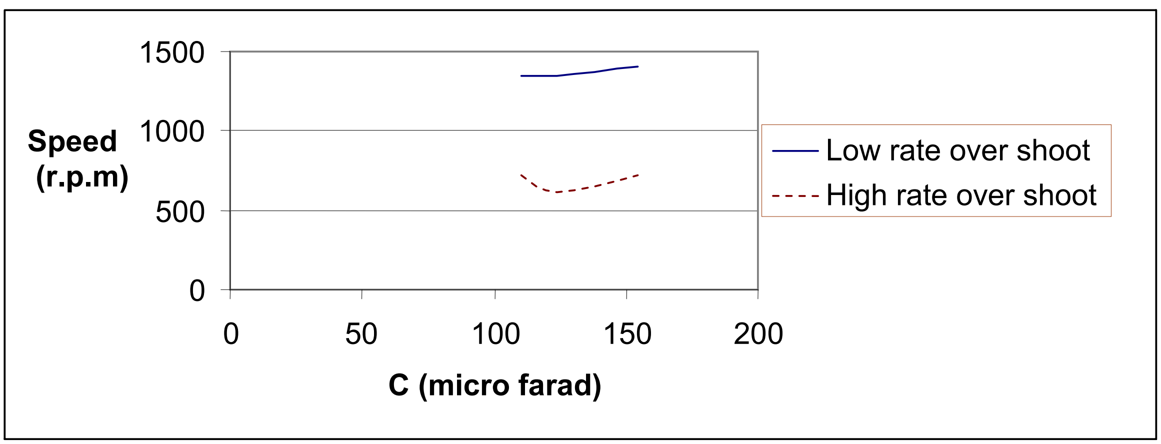


Figure (3.9)

Low/high rate over shoot

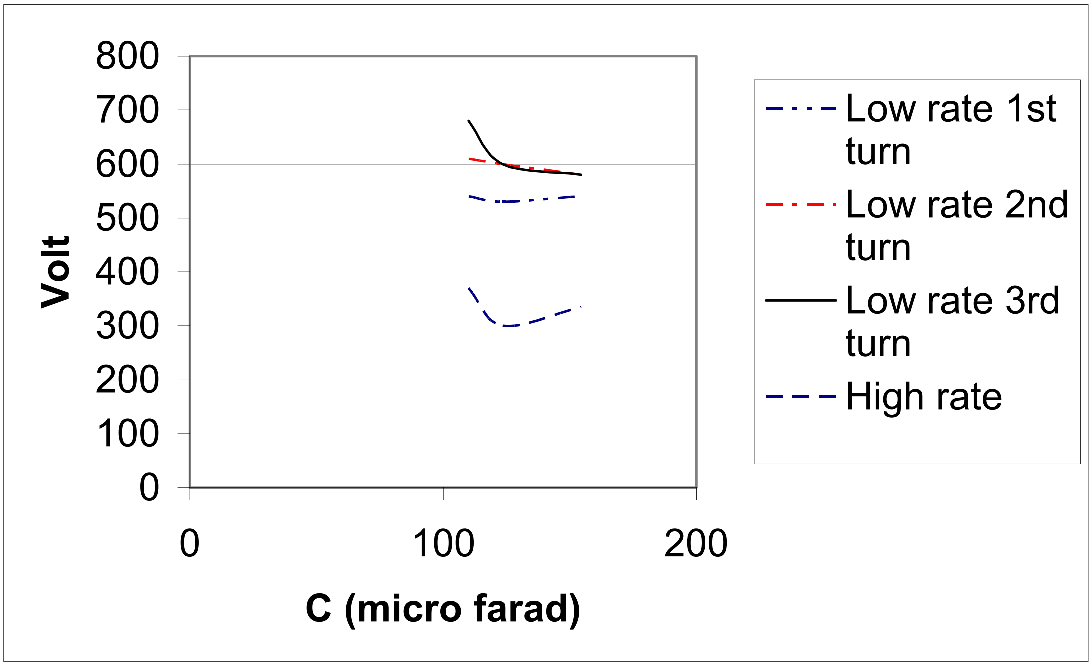


Figure (3.10)

Over shoot voltage pulse

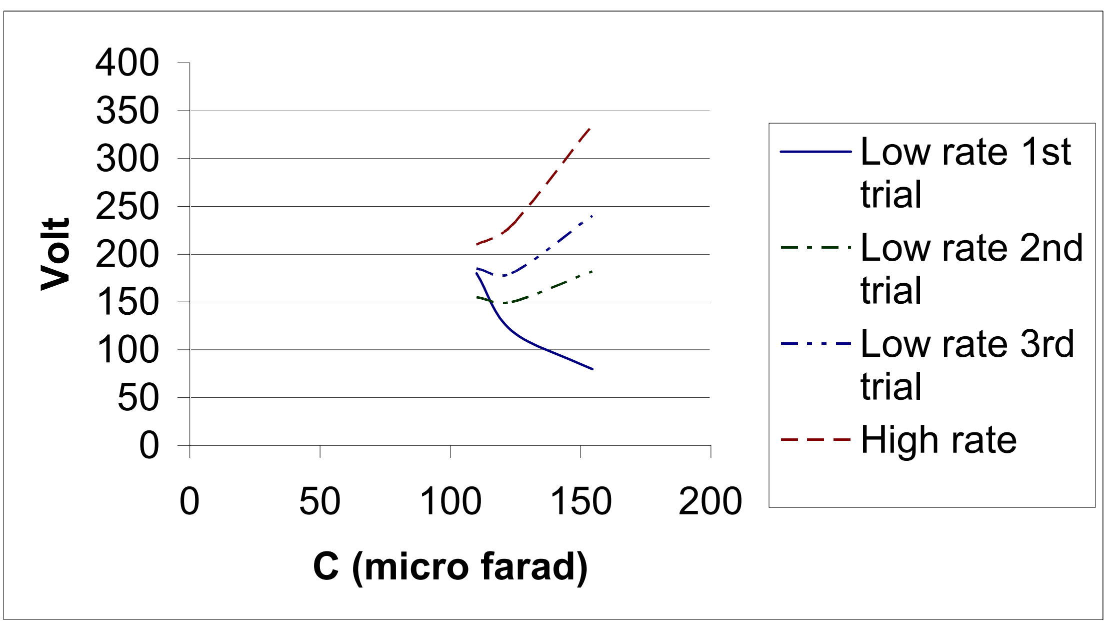


Figure (3.11)

Steady-State voltage

3.5 Low Acceleration Rate Voltage Build-up Process Experiment (No.4):

In this experiment, more detailed observations of possible excitation before the occurance of excitation speeds presented previously is conducted. This required digital measurements of voltages while the machine is accelerated at low rate. The result is that a define build up of voltage although of small magnitude accompanies excitation speeds. These are shown recorded in Figures (3.12, 13, 14, 15, 16, and 17) for different values of capacitors. Note the decrease in possible excitation speeds with increased capacitance. This general behavior is consistent in both the low rate and high rate excitation boundaries. Note also the decrease in both speed range and voltage amplitude with increased C.

At the end of each pulse the occurance of self-excitation is bound to happen so that this could be a good indication for the occurance of self-excitation in stand-by application where the machine could be revolving at the speed corresponding to zero voltage and readily available for excitation.

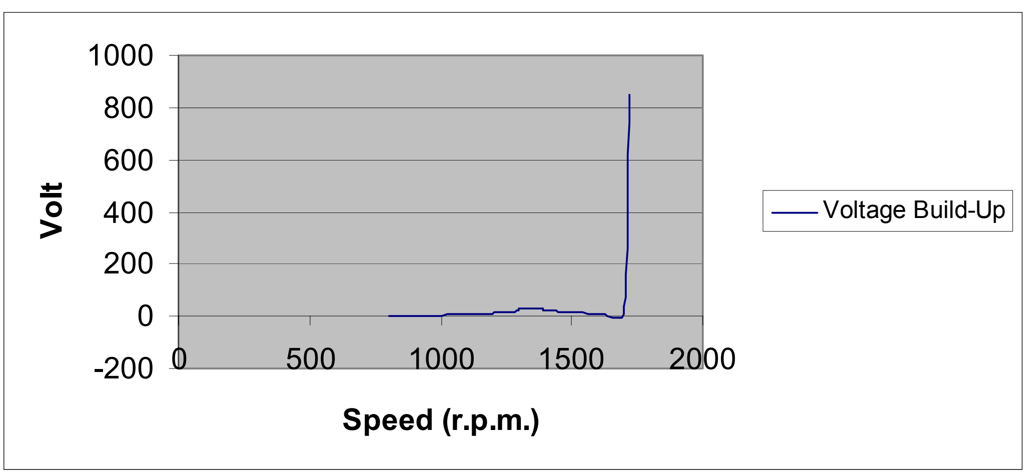


Figure (3.12)

Voltage Build-up with 20 Micro Farad External Capacitor

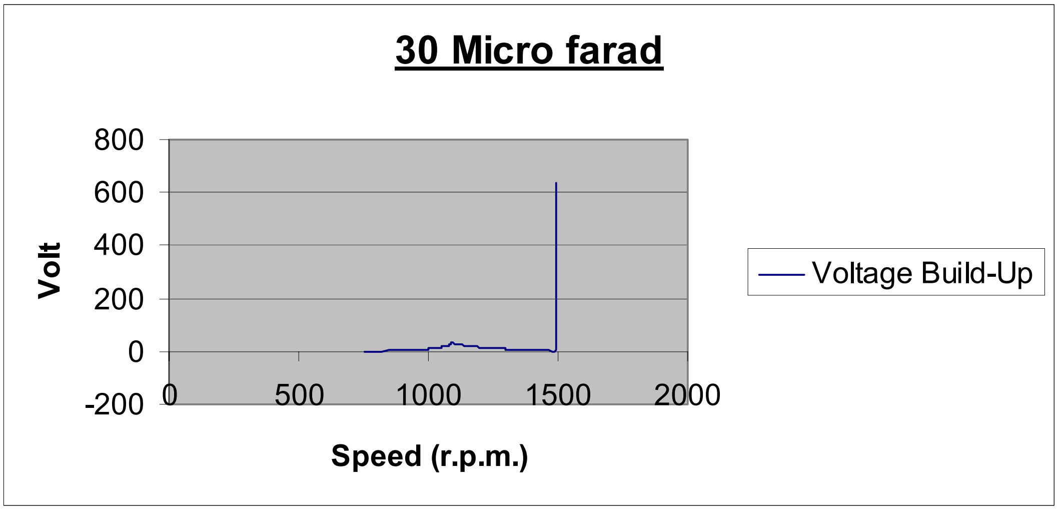


Figure (3.13)

Voltage Build-up with 30 Micro Farad External Capacitor

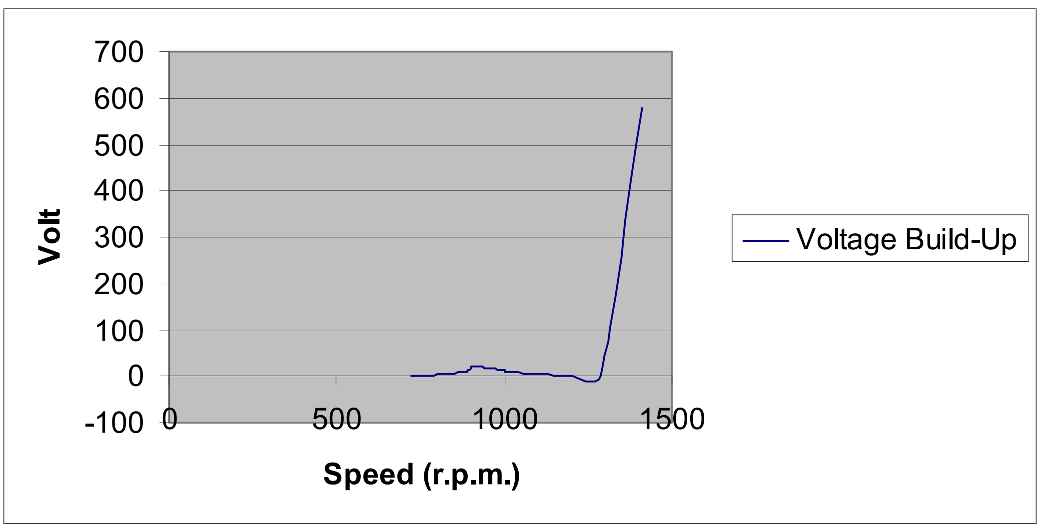


Figure (3.14)

Voltage Build-up with 45 Micro Farad External Capacitor

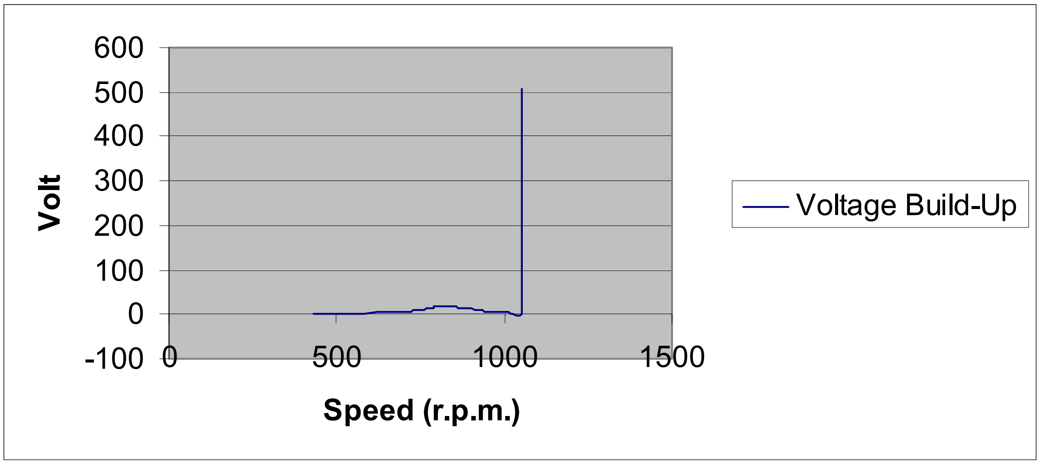


Figure (3.15)

Voltage Build-up with 50 Micro Farad External Capacitor

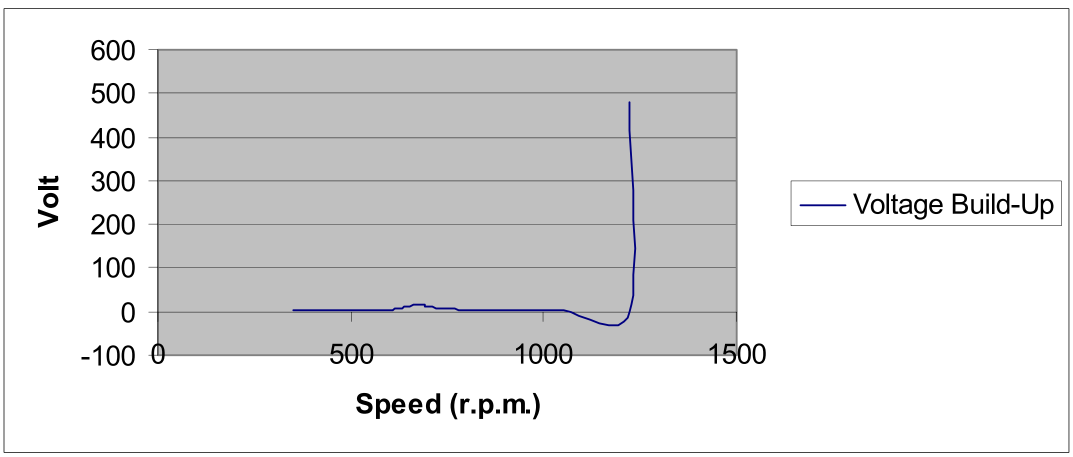


Figure (3.16)

Voltage Build-up with 85 Micro Farad External Capacitor

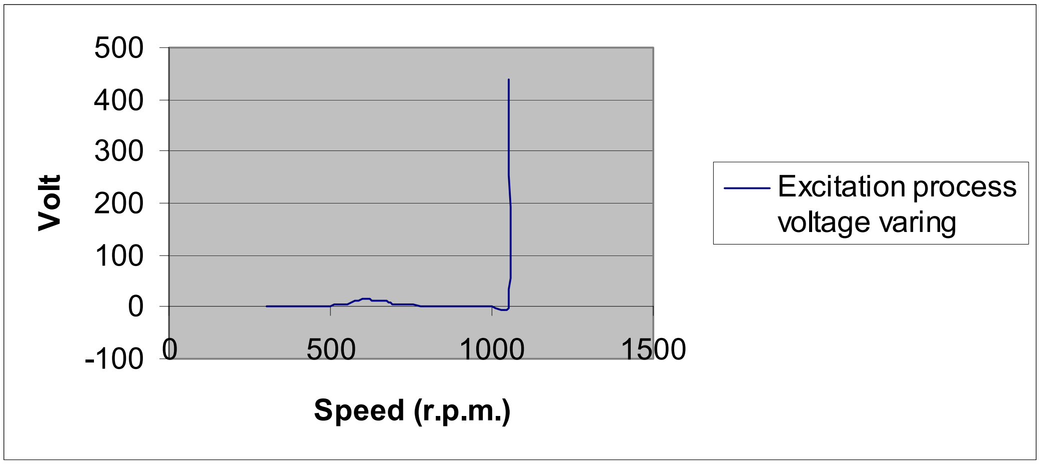


Figure (3.17)

Voltage Build-up with 100 Micro Farad External Capacitor

The following table shows comparisons on the voltage build-up characteristics while excitation process for different capacitances.

Table (3.6)

Transient Speed Range Characteristics

C (micro farad)	**Speed Range** (r.p.m.)	**Voltage amplitude** (Volt)
20	920	31
30	740	31
45	690	22
55	620	20
61	620	20
69	610	19
75	750	16
80	590	9
85	870	15
100	750	16
105	890	13

3.6 Summary of the Experimental Observations:

In addition to the steady-state excitation and cut-off speed boundaries reported in the literature for both low and high primemover speed rates, the number of experiments presented here gave additional information. This includes:

a) Over-shoot speeds where the machine would initially excite.
b) Over-shoot voltage pulse. And,
c) Possible excitation with a given voltage pulse around 50% of the final steady-state speed value. Further information obtained from this voltage pulse may be concluded by extrapolations to the final steady-state speed at the tangent of the rising portion of the pulse to give the first over-shoot voltage expected from a given machine capacitor set.

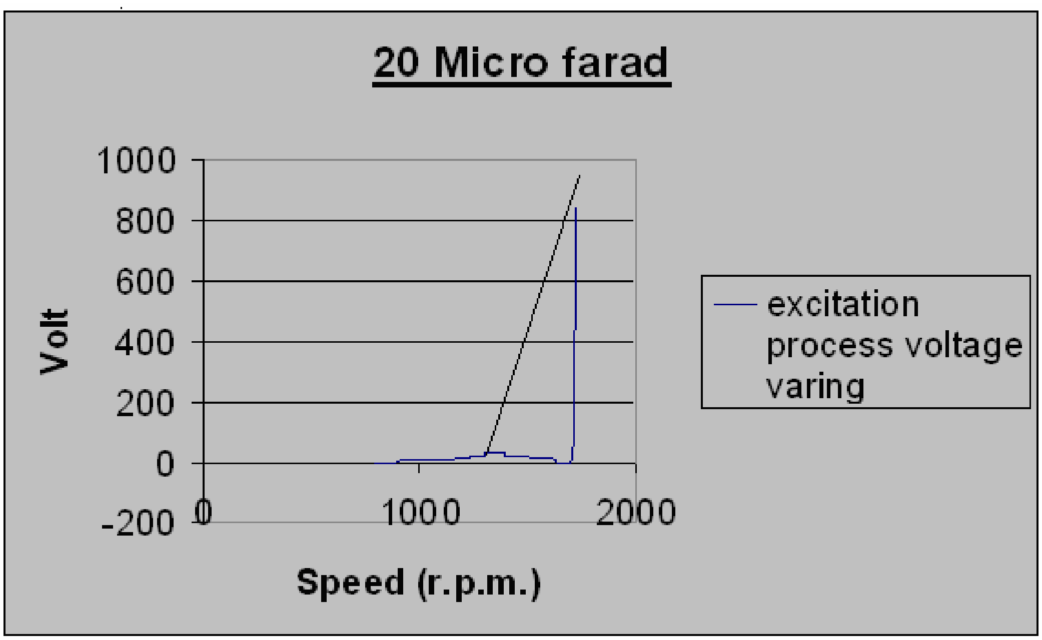


Figure (3.18)

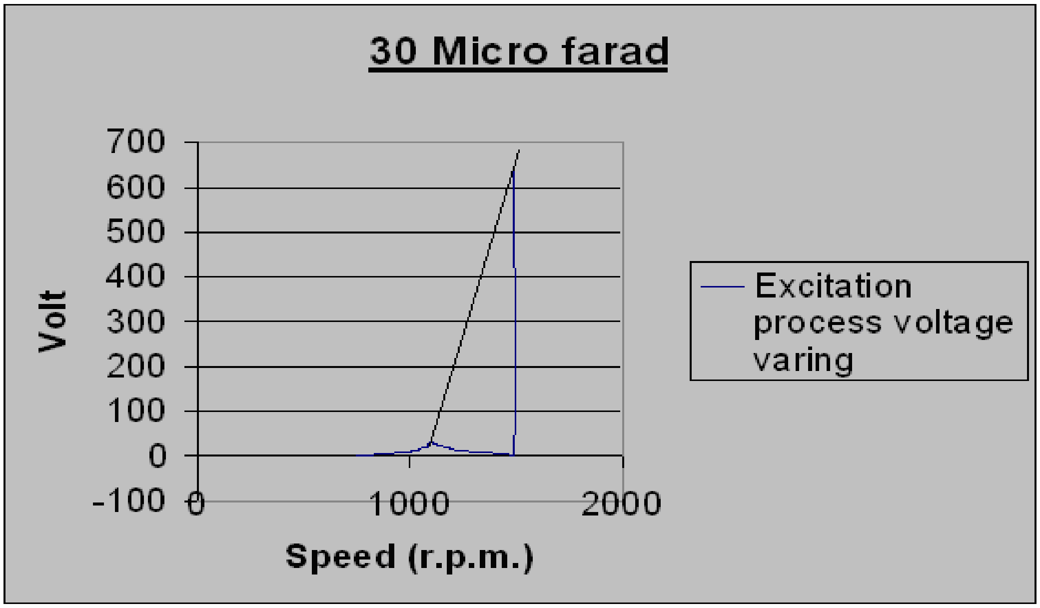


Figure (3.19)

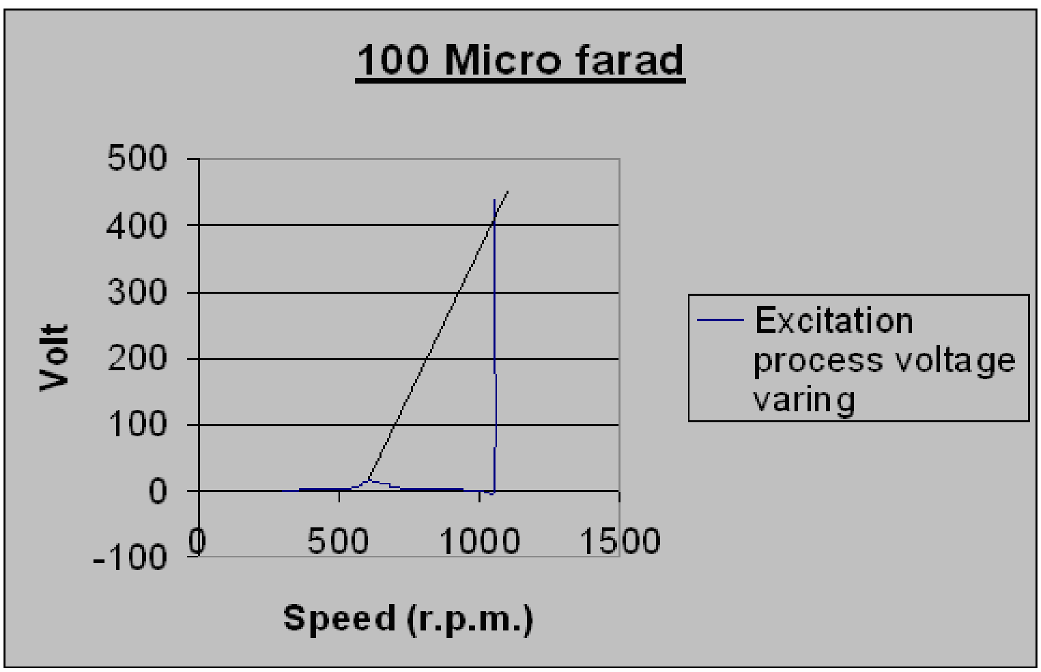


Figure (3.20)

The curves shown in Figures (3.18, 19, and 20) show the results of this experiment where the low-rate initial voltage over-shoot could be predicted.

Chapter Four

Attempted Numerical Solution to the SEIG System Equations

4.1 System Equations:

In this chapter, an attempt is made to find the roots of the equations derived from the per-phase equivalent circuit that correspond to the observed speeds of a SEIG system as described in the previous chapters. Referring to the per-phase equivalent circuit shown in Figure (2.1) in Chapter Two, the loop equations are:

$$[Z]\ [I] = 0 \quad \text{-----------------------------} \ (4.1)$$

Since $I \neq 0$, then Z which is the net loop impedance must be equal to zero. Hence;

$$\mathrm{Re}\ (Z) = \mathrm{Im}\ (Z) = 0 \ \text{--------------------} \ (4.2)$$

where,

$$Z = [((Rr/\ (N_s\text{-}v) + j\ X_{lr})\ //\ j\ X_m] + (Rs/\ N_s) + j\ Xls$$
$$+ [(-j\ X_c/\ N_s^2)\ //\ ((R/\ N_s) + j\ X)] \quad \text{-------------} \ (4.3)$$

The expansions of the real and imaginary parts of Z give the following results (see appendix A):

$$
\begin{aligned}
Re(Z) = {} & [2*X_m*X_{lr}*(N_S\text{-}v)\hat{}2*N_S\hat{}3]/[Rs*(Rr*R)\hat{}2] + \\
& [X\hat{}2*X_{lr}\hat{}2*(N_S\text{-}v)\hat{}2*N_S\hat{}7]/[Rs*(Rr*X_c*R)\hat{}2] + \\
& [X_{lr}\hat{}2*(N_S\text{-}v)\hat{}2* N_S\hat{}3]/[Rs*(Rr*R)\hat{}2]+ \\
& [2*X* N_S\hat{}5]/[X_c*R*(Rs)\hat{}2]+ \\
& [2*X*X_m\hat{}2*(N_S\text{-}v)\hat{}2* N_S\hat{}5]/[X_c *Rr*(Rs*R)\hat{}2]+ \\
& [4*X*X_m*Xlr*(N_S\text{-}v)\hat{}2* N_S\hat{}5]/[X_c*R*(Rs* Rr)\hat{}2]+ \\
& [2*X*X_{lr}\hat{}2*(N_S\text{-}v)\hat{}2* N_S\hat{}5]/[X_c*R*(Rs* Rr)\hat{}2]+ \\
& [X*X_m\hat{}2*(N_S\text{-}v)\hat{}2* N_S\hat{}3]/[R*(Rs* Rr)\hat{}2]+ \\
& [2*X*X_m*X_{lr}*(N_S\text{-}v)\hat{}2* N_S\hat{}3]/[R*(Rs* Rr)\hat{}2]+ \\
& [X*X_{lr}*(N_S\text{-}v)\hat{}2* N_S\hat{}3]/[R*(Rs* Rr)\hat{}2]- \\
& [2*X_m*X_{lr}*(N_S\text{-}v)* N_S\hat{}6]/[Rr*(Rs*X_c)\hat{}2]- \\
& [X_m\hat{}2*(N_S\text{-}v)* N_S\hat{}6]/[Rr *(Rs*X_c)\hat{}2]- \\
& [2*X\hat{}2*X_m*X_{lr}*(N_S\text{-}v)* N_S\hat{}8]/[Rr*(Rs*X_c*R)\hat{}2]- \\
& [2*X_m*X_{lr}*(N_S\text{-}v)* N_S\hat{}4]/[Rr*(Rs*R)\hat{}2]- \\
& [2*X*X_m\hat{}2*(N_S\text{-}v)* N_S\hat{}6]/[Rr*X_c*(Rs*R)\hat{}2]- \\
& [X\hat{}2* N_S\hat{}7]/[Rs*(R*X_c)\hat{}2]- \\
& [N_S\hat{}3]/[Rs*(R)\hat{}2]- \\
& [X_m\hat{}2*(N_S\text{-}v)\hat{}2*N_S\hat{}5]/[Rs*(Rr*X_c)\hat{}2]- \\
& [2*X_m*X_{lr}*(N_S\text{-}v)\hat{}2*N_S\hat{}5]/[Rs*(Rr*X_c)\hat{}2]- \\
& [X_{lr}\hat{}2*(N_S\text{-}v)\hat{}2*N_S\hat{}5]/[Rs*(Rr*X_c)\hat{}2]- \\
& [2*X*X_m\hat{}2*(N_S\text{-}v)\hat{}2*N_S\hat{}5]/[Rs*X_c*(Rr*R)\hat{}2]- \\
& [4*X*X_m*X_{lr}*(N_S\text{-}v)\hat{}2*N_S\hat{}5]/[Rs*X_c*(Rr*R)\hat{}2]- \\
& [2*X*X_{lr}*(N_S\text{-}v)\hat{}2*N_S\hat{}5]/[Rs*X_c*(Rr*R)\hat{}2]- \\
& [X*N_S\hat{}3]/[R*(Rs)\hat{}2] = 0 \qquad \text{------------------------------} \quad (4.4)
\end{aligned}
$$

$$
\begin{aligned}
Im(Z) = {} & (X_m*N_S\hat{}4)/((X_c)\hat{}2)+ \\
& (X_m\hat{}2*X_{lr}*(N_S\text{-}v)\hat{}2*N_S\hat{}4)/((Rr*X_c)\hat{}2)+ \\
& (X_m*X_{lr}\hat{}2*(N_S\text{-}v)\hat{}2*N_S\hat{}4)/((Rr*X_c)\hat{}2)+ \\
& (X_{ls}*N_S\hat{}4)/((Xc)\hat{}2)+ \\
& (X*N_S\hat{}2)/((R)\hat{}2)+ \\
& (2*X_m*X*N_S\hat{}4)/(X_c*(R)\hat{}2)+ \\
& (2*X_m\hat{}2*X_{lr}*X*(N_S\text{-}v)\hat{}2*N_S\hat{}4)/(X_c*(Rr*R)\hat{}2)+ \\
& (2*X_m*X_{lr}\hat{}2*X*(N_S\text{-}v)\hat{}2*N_S\hat{}4)/(X_c*(Rr*R)\hat{}2)+
\end{aligned}
$$

$(2*X_{ls}*X*N_S\hat{}4)/(X_c*(R)\hat{}2)+$

$(X_{ls}*X_m\hat{}2*X\hat{}2*(N_S-v)\hat{}2*N_S\hat{}6)/((Rr*R*X_c)\hat{}2)+$

$(2*X_{ls}*X_m*X_{lr}*X\hat{}2*(N_S-v)\hat{}2*N_S\hat{}6)/((Rr*R*X_c)\hat{}2)+$

$(X_{ls}*X_{lr}\hat{}2*X\hat{}2*(N_S-v)\hat{}2*N_S\hat{}6)/((Rr*R*X_c)\hat{}2)+$

$(X_{ls}*X_m\hat{}2*(N_S-v)\hat{}2*N_S\hat{}2)/((Rr*R)\hat{}2)+$

$(2*X_{ls}*X_m*X_{lr}*(N_S-v)\hat{}2*N_S\hat{}2)/((Rr*R)\hat{}2)+$

$(X_{ls}*X_{lr}\hat{}2*(N_S-v)\hat{}2*N_S\hat{}2)/((Rr*R)\hat{}2)+$

$(X_m\hat{}2*X\hat{}2*(N_S-v)\hat{}2*N_S\hat{}4)/(X_c*(Rr*R)\hat{}2)+$

$(2*X_m*X_{lr}*X\hat{}2*(N_S-v)\hat{}2*N_S\hat{}4)/(X_c*(Rr*R)\hat{}2)+$

$(X_{lr}\hat{}2*X\hat{}2*(N_S-v)\hat{}2*N_S\hat{}4)/(X_c*(Rr*R)\hat{}2)+$

$(X_m\hat{}2*(N_S-v)\hat{}2*N_S\hat{}2)/(X_c*(Rr)\hat{}2)+$

$(2*X_m*X_{lr}*(N_S-v)\hat{}2*N_S\hat{}2)/(X_c*(Rr)\hat{}2)+$

$(X_{lr}\hat{}2*(N_S-v)\hat{}2*N_S\hat{}2)/(X_c*(Rr)\hat{}2)-$

$(X\hat{}2*N_S\hat{}4)/(X_c*(R)\hat{}2)-$

$(N_S\hat{}2)/((X_c)\hat{}2)-$

$(X_m\hat{}2*X_{lr}*(N_S-v)\hat{}2*N_S\hat{}2)/((Rr*R)\hat{}2)-$

$(X_m*X\hat{}2*N_S\hat{}6)/((R*X_c)\hat{}2)-$

$(X_m*N_S\hat{}2)/((R)\hat{}2)-$

$(X_m\hat{}2*X_{lr}*X\hat{}2*(N_S-v)\hat{}2*N_S\hat{}6)/((Rr*R*X_c)\hat{}2)-$

$(X_m*X_{lr}\hat{}2*X\hat{}2*(N_S-v)\hat{}2*N_S\hat{}6)/((Rr*R*X_c)\hat{}2)-$

$(X_m*X_{lr}\hat{}2*(N_S-v)\hat{}2*N_S\hat{}2)/((Rr*R)\hat{}2)-$

$(X_{ls}*X\hat{}2*N_S\hat{}6)/((R*X_c)\hat{}2)-$

$(X_{ls}*N_S\hat{}2)/((R)\hat{}2)-$

$(X_{ls}*X_m\hat{}2*(N_S-v)\hat{}2*N_S\hat{}4)/((Rr*X_c)\hat{}2)-$

$(2*X_{ls}*X_m*X_{lr}*(N_S-v)\hat{}2*N_S\hat{}4)/((Rr*X_c)\hat{}2)-$

$(X_{ls}*X_{lr}\hat{}2*(N_S-v)\hat{}2*N_S\hat{}4)/((Rr*X_c)\hat{}2)-$

$(2*X_{ls}*X_m\hat{}2*(N_S-v)\hat{}2*N_S\hat{}4)/(X_c*(Rr*R)\hat{}2)-$

$(4*X_{ls}*X_m*X_{lr}*X*(N_S-v)\hat{}2*N_S\hat{}4)/(X_c*(Rr*R)\hat{}2)-$

$(2*X_{ls}*X_{lr}\hat{}2*X*(N_S-v)\hat{}2*N_S\hat{}4)/(X_c*(Rr*R)\hat{}2)-$

$(X_m\hat{}2*X*(N_S-v)\hat{}2*N_S\hat{}2)/((Rr*R)\hat{}2)-$

$(2*X_m*X_{lr}*X*(N_S-v)\hat{}2*N_S\hat{}2)/((Rr*R)\hat{}2)-$

$(2*X_{lr}\hat{}2*X*(N_S-v)\hat{}2*N_S\hat{}2)/((Rr*R)\hat{}2) = 0$ -------------- (4.5)

4.2 Solution Algorithm:

The two equations stated above are of order 6, 2, 2 in terms of N_S, X_c, and v respectively.

With the aid of Matlab (version 6.5) symbolic features, solutions had been obtained for X_c in terms of (N_S, v) and for v in terms of (N_S, X_c) (see appendix B where X_c is substituted by C for software formulation).

The solution algorithm shown in Figure (4.1) starts by assuming all other coefficients as constants and for a given v =1.0 p.u. the two equations from Real and Imaginary parts in X_c are plotted for varying N_S and their intersection gives the required capacitance and the corresponding N_S of self-excitation.

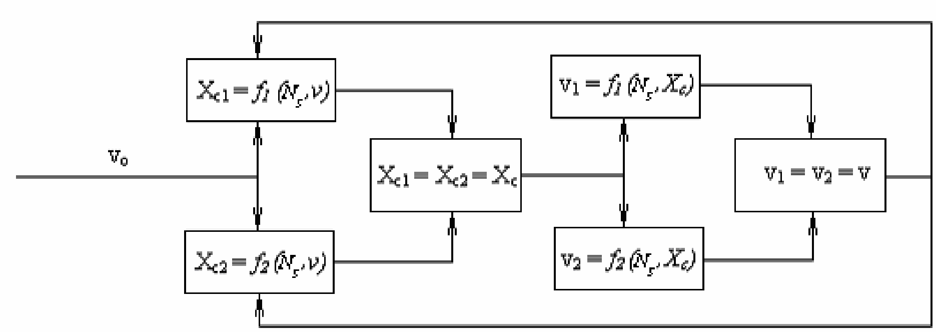


Figure (4.1)

Solution Algorithm of System Equations

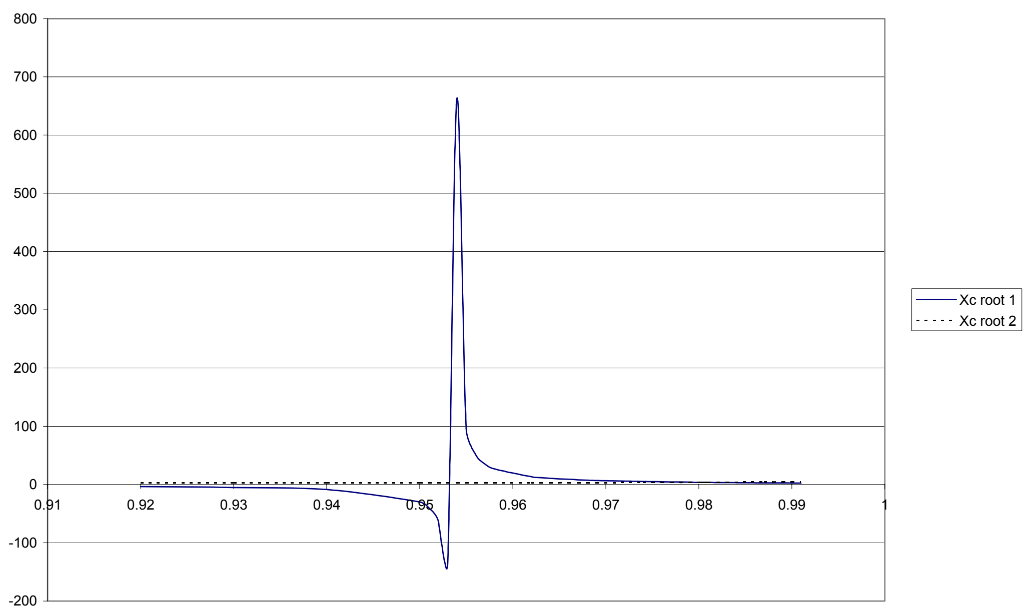
This method had been applied to the example given in Section (2.2.4) and for the same machine the minimum excitation capacitor is found to be 33 µ.F. (X_c = 3.6934 p.u.). Table (4.1) shows the results of this method of calculation; obtained from plots shown in Figure (4.2) – (a) and (b).

Table (4.1)

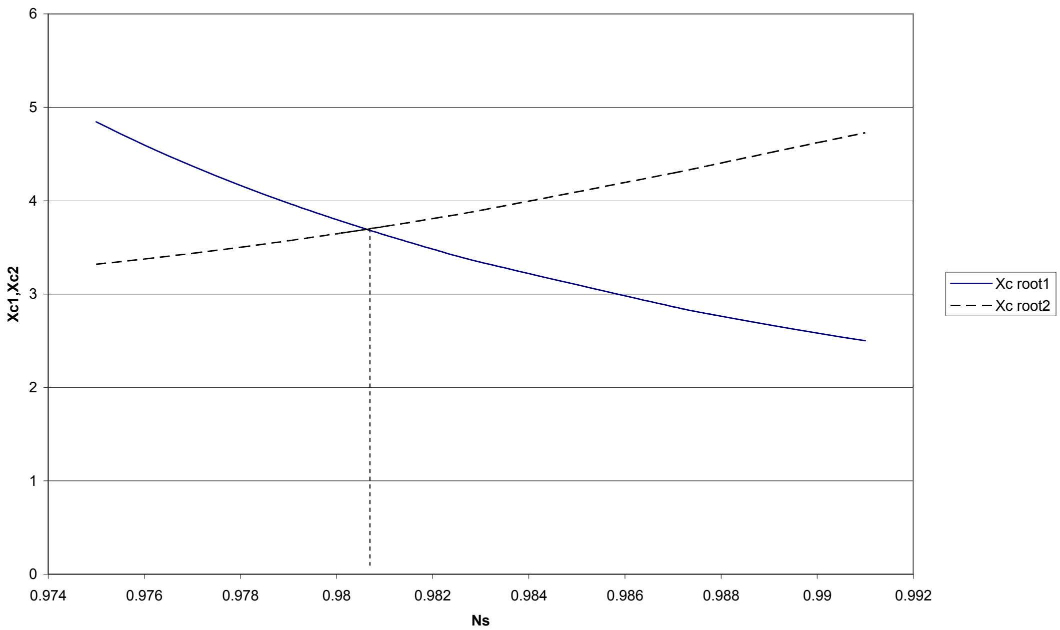
Numerical method results

N_s	X_{c1}	X_{c2}
0.92	-3.5857	2.4005
0.93	-5.0252	2.4658
0.94	-8.4974	2.5454
0.95	-29.8966	2.6513
0.951	-40.3393	2.6642
0.952	-62.1947	2.6776
0.953	-136.755	2.6916
0.954	661.5217	2.7062
0.955	96.187	2.7215
0.956	51.6931	2.7376
0.957	35.2605	2.7544
0.958	26.7051	2.7722
0.962	13.4117	2.8535
0.963	11.8973	2.8769
0.964	10.6796	2.9018
0.967	8.1308	2.9866
0.968	7.5195	3.0187
0.969	6.988	3.0532
0.974	5.113	3.2665
0.975	4.8429	3.3191
0.976	4.5968	3.3756
0.977	4.3715	3.4362
0.978	4.1646	3.5012
0.979	3.974	3.5708
0.98	3.7977	3.6452
0.9801	3.7808	3.6529
0.9802	3.7641	3.6606
0.9803	3.7474	3.6684
0.9804	3.7309	3.6763
0.9805	3.7145	3.6842
0.9806	3.6982	3.6921

0.98061	3.6966	3.6929
0.98062	3.695	3.6937
0.98063	3.6934	3.6945
0.98064	3.6918	3.6953
0.98065	3.6902	3.6961
0.9807	3.6821	3.7001
0.9808	3.6661	3.7082
0.9809	3.6501	3.7163
0.981	3.6344	3.7244
0.982	3.4826	3.8085
0.983	3.3413	3.8974
0.987	2.8641	4.296
0.988	2.7636	4.4034
0.99	2.5821	4.6211
0.991	2.5004	4.7285



(a)



(b)

Figure (4.2)

X_c Roots Intersection

The resultant X_c ($X_{c1} = X_{c2}$) is then treated as a fixed parameter for the two equations in v_1 and v_2 which for varying N_s gave an intersection v to be used in the second trial for X_c equations. This procedure which is shown in the Figure is repeated as long as $N_s < v$ which is the condition for self-excitation. These results are calculated for the parameters in Ref. [1] and shown in Table (4.2).

The table shows the existence of a lower limit to X_c below which there could be no self-excitation.

Table (4.2)

Dependant Solutions' Results

N_s	Xc	*v*	C
0.98062	3.6934	1	0.0000331
0.934	3.6934	0.9187	0.0000331
0.8993	3.1805	0.9187	0.0000385
0.82699	3.1805	0.8464	0.0000385
0.798	2.751	0.8464	0.0000445
0.762604	2.751	0.782	0.0000445
0.7405	2.389	0.782	0.0000512
0.704855	2.389	0.7242	0.0000512
0.6885	2.0795	0.7242	0.0000589
0.6885	2.0795	0.6719	0.0000589
0.652663	1.8104	0.6719	0.0000676
0.64155	1.8104	0.6247	0.0000676

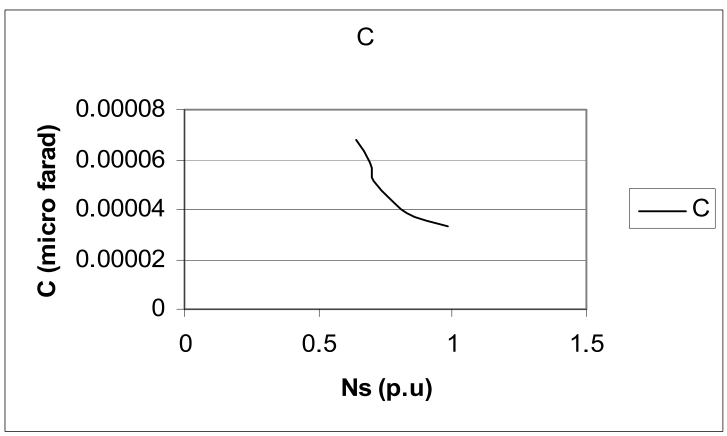


Figure (4.3)

Minimum Capacitor Depending on Rotor Speed

4.3 Searching for the Possible Pre-Nominal Excitation Mode:

In order to explain the possible pre-nominal excitation mode shown in Figure (4.4) as observed experimentally in Chapter Three, the machine reactance X_m was treated as the dependant variable for a given value of X_c that would cause self-excitation at v = 1.0.

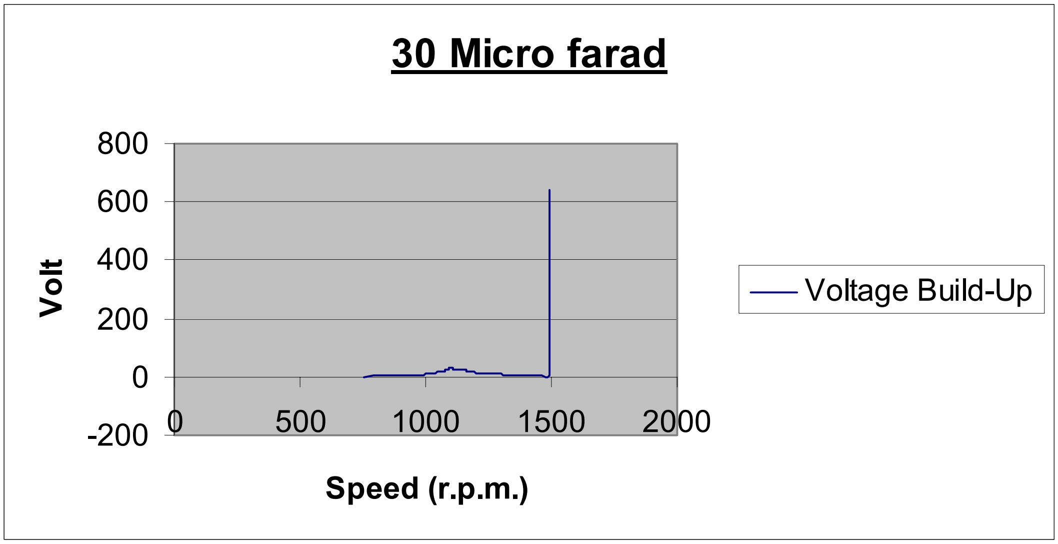
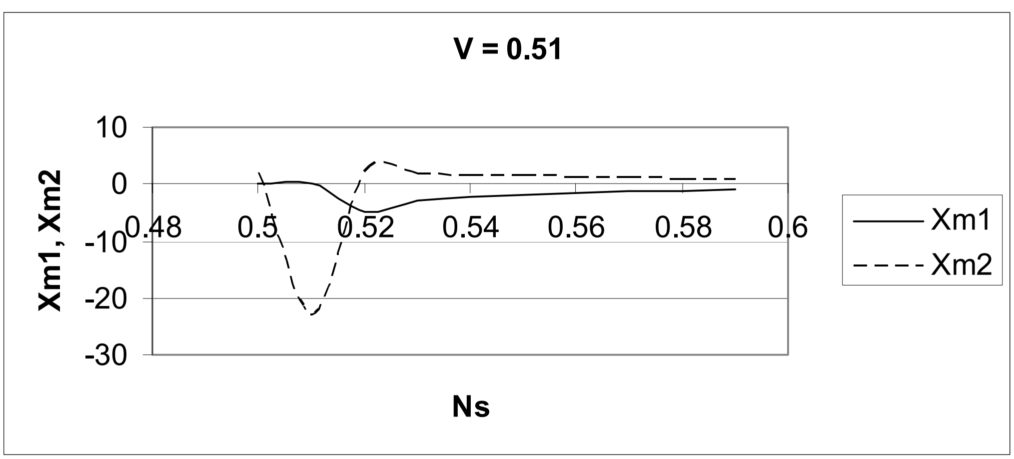


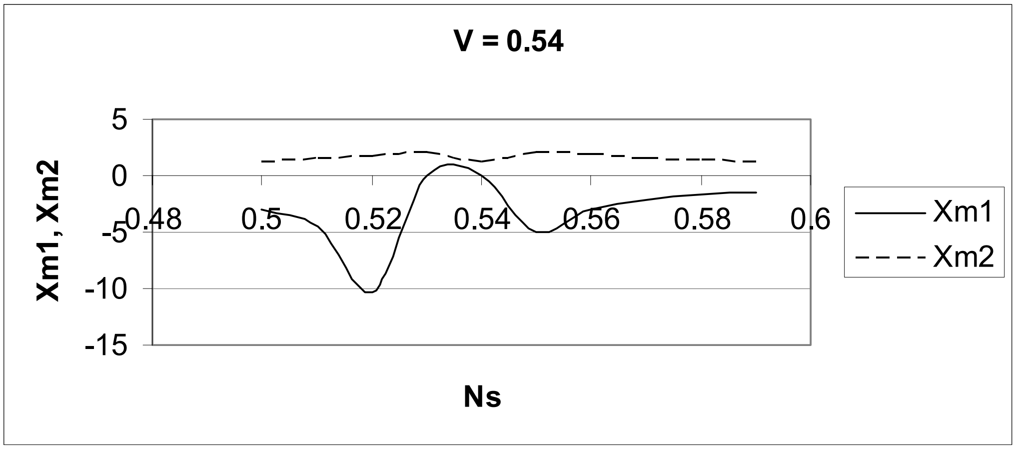
Figure (4.4)

Voltage Build-Up While Exciting Process

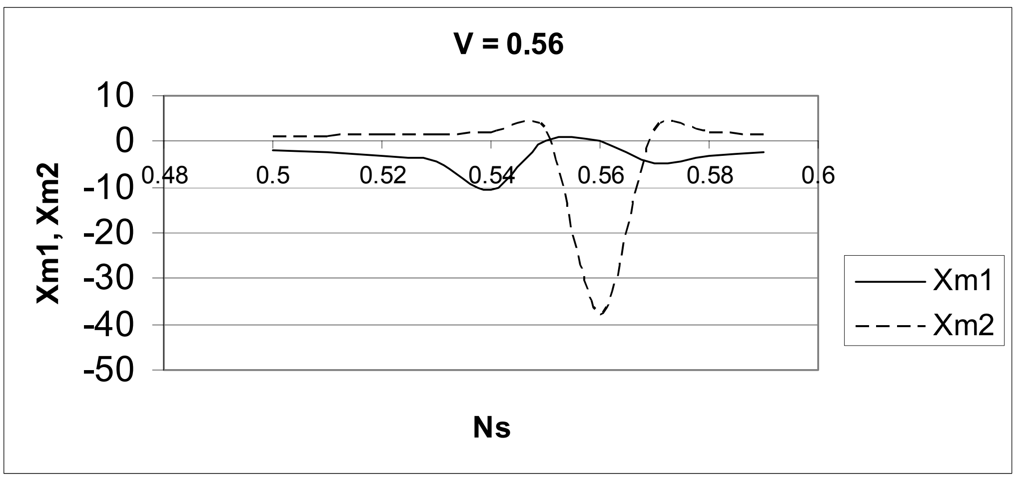
The search consists of varying F for a given v around the observed lower range and looking for the two roots of X_m that would coincide. The results of this search are shown in Figure (4.5) where it can be seen that a possible excitation at v = 0.54 could occur although a definite intersection was not found.



(a)



(b)



(c)

Figure (4.5)

Searching for the Possible Pre-Nominal Excitation Speed

4.4 Theoretical Verification of the No-load Experimental Results:

Instead of treating the no-load case as a special case of $R_L \rightarrow \infty$ as given in Reference [1], the no-load equations are derived directly from Figure (4.6) as follows:

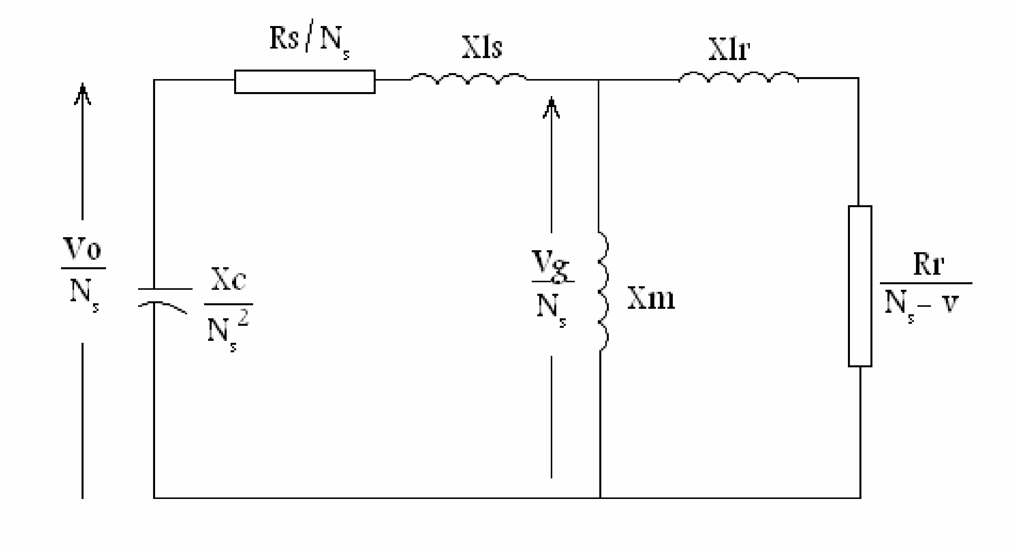


Figure (4.6)

No-load Equivalent Circuit Model

$$Re\,(Z) = (1/C)-(((X_m * X1+ X_m*X2+X1*X2)/(R1*R2*C))*(N_S-v)*N_S) = 0 \text{ ----- } (4.6)$$

$$Im\,(Z) = ((X_m*N_S\text{^}2)/(R2*C))+((X_m+X1)*(N_S-v)*N_S/(R1*C))+ ((X2*N_S\text{^}2)/(R2*C))-(1/R2) = 0 \text{ ----- } (4.7)$$

These two equations had been solved with the computer aid just as the previous section.

In this test the parameters of the machine which was used in the experiments presented in Chapter Three had been applied to the no-load equivalent circuit equations (see appendix D). Table (4.4) and Figure (4.7) show the comparison of the experimental readings with the numerical solution obtained. As it appears the results of the equations give close values to some of the experimental ones.

Table (4.4)

A comparison of Experimental and Numerical Results

N_S r.p.m	C Experimental	C Theoretical	C Theoretical (p.u.)	v (p.u.)	N_S (p.u.)
2250		0.000021	15.535 p.u.	1.5	1.5028
1740	0.00002	0.000033	9.985 p.u.	1.2	1.2035
1490	0.00003	0.000047	6.97 p.u.	1	1.0041
1410	0.000045	0.000053	6.174 p.u.	0.94	0.9444
1350	0.00011	0.000058	5.67 p.u.	0.9	0.9046
1180	0.000075	0.000073	4.505 p.u.	0.8	0.8052
1120	0.00008	0.000083	3.938 p.u.	0.7466	0.7521
1050	0.0001	0.000095	3.476 p.u.	0.7	0.7059
900		0.000127	2.587 p.u.	0.6	0.6069
750		0.000179	1.832 p.u.	0.5	0.5082

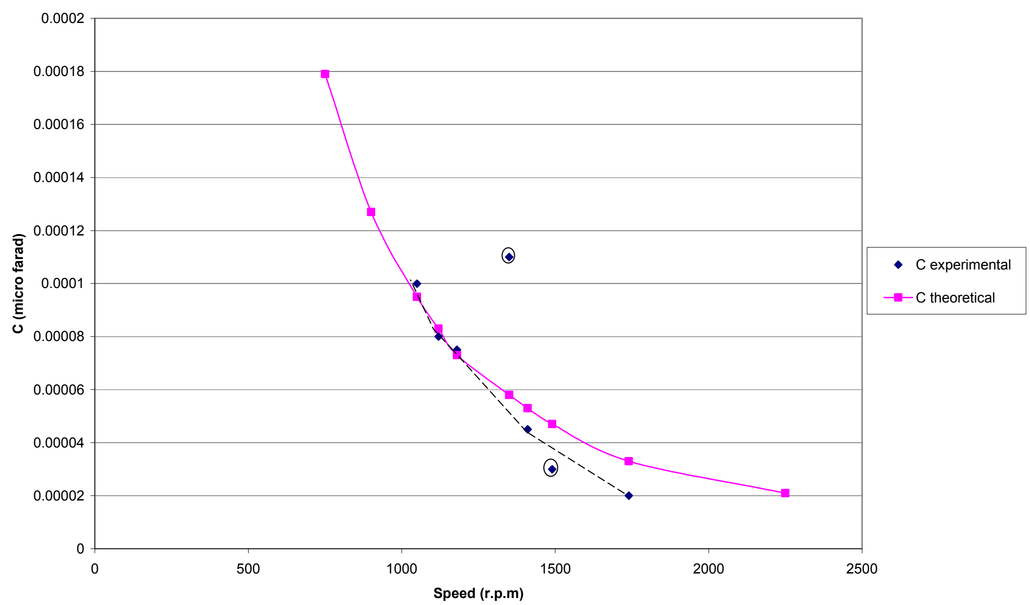


Figure (4.7)

A comparison of Experimental and Numerical Results

4.5 Conclusions:

The search for the minimum excitation capacitance at a given speed and the possible pre-nominal excitation speed for a given capacitance has been conducted using two solution algorithms of the two machine equations. The first gave the typical low rate capacitance – speed boundary. The second in which was treated in terms of machine X_m gave no direct solution to the pre-nominal speed but the possibility of the existence of solution has been shown.

Chapter Five

Conclusions

The project has presented experimental observations of the behaviour of a capacitor-excited induction generator driven on no-load at two different ramp-rates with capacitors permanently connected.

The system equations are non-linear and different numerical methods for solving these equations have been used. The observation of a voltage pulse signaling the possible existence of an excitation speed much lower than the nominal value for a given capacitance has prompted the suggestion of a different solution algorithm to system equations. While no definite conclusions have appeared from these algorithms, it is suggested that a more detailed investigation be carried out about the existence of this pre-nominal speed.

In addition to the steady-state excitation and cut-off speed boundaries reported in the literature for both low and high primemover speed rates, the number of experiments presented in this thesis gave additional information. This includes:

a) Over-shoot speeds where the machine would initially excite.
b) Over-shoot voltage pulse.
c) Possible excitation with a given voltage pulse around 50% of the final steady-state speed value. Further information obtained from this voltage pulse may be concluded by extrapolations to the final steady-state speed at the tangent of the rising portion of the pulse to give the first over-shoot voltage expected from a given machine capacitor set.

Appendix A
Solution of the Characteristic Equation Of The Self-excited Machine

The equation governs the capacitor-excited machine as shown in chapters (2) and (4) is:

$$Z = [\ ((Rr/(N_S\text{-}v)) + j\ Xlr)\ //\ j\ Xm] + (Rs/N_S) + j\ Xls + [\ (-j\ Xc/N_S^2)\ //\ ((R/N_S) + j\ X)]$$

Let: -

$F = N_S$

$a = Rr\ /\ (F\text{-}v)$

$b = Rs\ /\ F$

$c = Xc\ /\ F^2$

$d = R\ /\ F$

➔ $Z = [\ (a + j\ Xlr)\ //\ j\ Xm] + b + j\ Xls + [\ -jc\ //\ (d + j\ X)]$

➔ $Z = [(j\ a\ Xm - Xlr\ Xm)/(a + j\ (Xm + Xlr))] + b + j\ Xls + [(c\ X - j\ c\ d)/(d + j\ (X\text{-}c))]$

$= [(-\ a\ Xlr\ Xm - a\ Xm\ (Xlr + Xm))\ /\ (a^2 - (Xm + Xlr)^2)] + b + [(c\ d\ X + c\ d\ (X\text{-}c)\ /\ (d^2 - (X\text{-}c)^2)] + j\ [\ ((a^2\ Xm + Xlr\ Xm\ (Xlr + Xm))\ /\ (a^2 - (Xlr + Xm)^2) + Xls - ((\ c\ X\ (X - c) + c\ d^2)\ /\ (d^2 - (X - c)^2))]$

➔ $Re(Z) = [(-\ a\ Xlr\ Xm - a\ Xm\ (Xlr + Xm))\ /\ (a^2 - (Xm + Xlr)^2)] + b + [(c\ d\ X + c\ d\ (X\text{-}c)\ /\ (d^2 - (X\text{-}c)^2)]$

& $Im(Z) = [\ ((a^2\ Xm + Xlr\ Xm\ (Xlr + Xm))\ /\ (a^2 - (Xlr + Xm)^2) + Xls - ((\ c\ X\ (X - c) + c\ d^2)\ /\ (d^2 - (X - c)^2))]$

For the existence of currents i.e. $I \neq 0$, Z must equal zero.
So, $Re(Z) = Im(Z) = 0$

$$\begin{aligned}
Re\ (Z) &= [(-\ a\ Xlr\ Xm - a\ Xm\ (Xlr + Xm))\ /\ (a^2 - (Xm + Xlr)^2)] + b + [(c\ d\ X + c\ d\ (X\text{-}c)\ /\ (d^2 - (X\text{-}c)^2)] \\
&= (\ (-\ 2\ a\ Xlr\ Xm - a\ (Xm)^2/\ (a^2 - (Xm + Xlr)^2)) + b + ((\ 2\ c\ d\ X - c^2\ d\ X)\ /\ (d^2 - (X\text{-}c)^2)) \\
&= [(\ (-\ 2\ a\ Xlr\ Xm - a\ (Xm)^2)\ (d^2 - (X\text{-}c)^2) + b\ (a^2 - (Xm + Xlr)^2))\ \ (d^2 - (X\text{-}c)^2) + ((\ 2\ c\ d\ X - c^2\ d\ X)\ (a^2 - (Xm + Xlr)^2))]\ /(a^2 - (Xm + Xlr)^2))\ (d^2 - (X\text{-}c)^2)) \\
&= 0
\end{aligned}$$

➔ $[(\ (-\ 2\ a\ Xlr\ Xm - a\ (Xm)^2)\ (d^2 - (X\text{-}c)^2) + b\ (a^2 - (Xm + Xlr)^2))\ (d^2 - (X\text{-}c)^2)$

$+ ((2 c d X - c^2 d X) (a^2 - (Xm + Xlr)^2))] = 0$ -------------------------------- (1)

$Im (Z) = [((a^2 Xm + Xlr Xm (Xlr + Xm)) / (a^2 - (Xlr + Xm)^2) + Xls – ((c X (X - c) + c d^2) / (d^2 - (X – c)^2))]$

$= ((a^2 Xm + (Xm)^2 Xlr + Xm (Xlr)^2) / (a^2 - (Xm + Xlr)^2)) + Xls – ((c X^2 - c^2 X + c d^2) / (d^2 - (X – c)^2))$

$= [(a^2 Xm + (Xm)^2 Xlr + Xm (Xlr)^2) (d^2 - (X – c)^2) + Xls (a^2 - (Xm + Xlr)^2) (d^2 - (X – c)^2) – ((c X^2 - c^2 X + c d^2)(a^2 - (Xm + Xlr)^2)] / (a^2 - (Xm + Xlr)^2) (d^2 - (X – c)^2))$

$= 0$

$\Rightarrow [(a^2 Xm + (Xm)^2 Xlr + Xm (Xlr)^2) (d^2 - (X – c)^2) + Xls (a^2 - (Xm + Xlr)^2) (d^2 - (X – c)^2) – ((c X^2 - c^2 X + c d^2)(a^2 - (Xm + Xlr)^2)] = 0$ -------------------------------- (2)

Dividing both Real and Imaginary parts by $(a b c d)^2$, and making the following substitution for matlab formulation :

Rr = R1
Rs = R2
Xlr = X1
Xls = X2
Xm = M
Xc = C

We get after elaborate manipulation the following expressions. These could represent polynomials in either F, C or v.

Real Part: -

```
(X^2*M^2*F^6-(X^2*M^2*F^5*v))/(R1*(R2*C*R)^2)+
(X^2*M^2*F^6-(X^2*M^2*F^5*2*v)+X^2*M^2*F^4*v^2)/(R2*(R1*C*R)^2)+
(2*X^2*M*X1*F^6-
(2*X^2*M*X1*F^5*2*v)+2*X^2*M*X1*F^4*v^2)/(R2*(R1*C*R)^2)+
(X^2*X1^2*F^6-(X^2*X1^2*F^5*2*v)+X^2*X1^2*F^4*v^2)/(R2*(R1*C*R)^2)-
(2*X^2*M*X1*F^6-(2*X^2*M*X1*F^5*v))/(R1*(R2*C*R)^2)+
(4*X*M*X1*F^4-(4*X*M*X1*F^3*v))/(R1*C*(R2*R)^2)-
(2*M*X1*F^4-(2*M*X1*F^3*v))/(R1*(R2*C)^2)-
(M^2*F^4-(M^2*F^3*v))/(R1*(R2*C)^2)-
(2*X*M^2*F^4-(2*X*M^2*F^3*v))/(R1*C*(R2*R)^2)+
(2*X*M^2*F^4-(2*X*M^2*F^3*2*v)+(2*X*M^2*F^2*v^2))/(C*R1*(R2*R)^2)+
(4*X*M*X1*F^4-(4*X*M*X1*F^3*2*v)+(4*X*M*X1*F^2*v^2))/(C*R*(R2*R1)^2)+
(2*X*X1^2*F^4-(2*X*X1^2*F^3*2*v)+(2*X*X1^2*F^2*v^2))/(C*R*(R2*R1)^2)-
(X^2*F^4)/(R2*(R*C)^2)-(M^2*F^4-
(M^2*F^3*2*v)+(M^2*F^2*v^2))/(R2*(R1*C)^2)-
(2*M*X1*F^4-(2*M*X1*F^3*2*v)+(2*M*X1*F^2*v^2))/(R2*(R1*C)^2)-(X1^2*F^4-
(X1^2*F^3*2*v)+(X1^2*F^2*v^2))/(R2*(R1*C)^2)-
(2*X*M^2*F^4-(2*X*M^2*F^3*2*v)+(2*X*M^2*F^2*v^2))/(R2*C*(R1*R)^2)-
(4*X*M*X1*F^4-(4*X*M*X1*F^3*2*v)+(4*X*M*X1*F^2*v^2))/(R2*C*(R1*R)^2)-
(2*X*X1*F^4-(2*X*X1*F^3*2*v)+(2*X*X1*F^2*v^2))/(R2*C*(R1*R)^2)+
(M^2*F^2-(M^2*F*v))/(R1*(R2*R)^2)+(F^2)/(R2*(C)^2)+
```

```
(2*X*F^2)/(R2*C*(R)^2)+
(M^2*F^2-(M^2*2*v*F)+(M^2*v^2))/(R2*(R1*R)^2)+
(2*M*X1*F^2-(2*M*X1*2*v*F)+(2*M*X1*v^2))/(R2*(R1*R)^2)+
(X1^2*F^2-(X1^2*2*v*F)+(X1^2*v^2))/(R2*(R1*R)^2)+
(2*X*F^2)/(C*R*(R2)^2)+
(X*M^2*F^2-(X*M^2*2*v*F)+(X*M^2*v^2))/(R*(R2*R1)^2)+
(2*X*M*X1*F^2-(2*X*M*X1*2*v*F)+(2*X*M*X1*v^2))/(R*(R2*R1)^2)+
(X*X1*F^2-(X*X1*2*v*F)+(X*X1*v^2))/(R*(R2*R1)^2)-
(2*M*X1*F^2-(2*M*X1*F*v))/(R1*(R2*R)^2)-
1/(R2*(R)^2)-
(X)/(R*(R2)^2)=0
```

Imaginary Part: -

```
(X2*M^2*X^2*F^6-
(X2*M^2*X^2*F^5*2*v)+(X2*M^2*X^2*F^4*v^2))/((R1*R*C)^2)+
(2*X2*M*X1*X^2*F^6-
(2*X2*M*X1*X^2*F^5*2*v)+(2*X2*M*X1*X^2*F^4*v^2))/((R1*R*C)^2)+
(X2*X1^2*X^2*F^6-
(X2*X1^2*X^2*F^5*2*v)+(X2*X1^2*X^2*F^4*v^2))/((R1*R*C)^2)-
(M^2*X1*X^2*F^6-
(M^2*X1*X^2*F^5*2*v)+(M^2*X1*X^2*F^4*v^2))/((R1*R*C)^2)-
(M*X1^2*X^2*F^6-
(M*X1^2*X^2*F^5*2*v)+(M*X1^2*X^2*F^4*v^2))/((R1*R*C)^2)+
(M^2*X1*F^4-(M^2*X1*F^3*2*v)+(M^2*X1*F^2*v^2))/((R1*C)^2)+
(M*X1^2*F^4-(M*X1^2*F^3*2*v)+(M*X1^2*F^2*v^2))/((R1*C)^2)+
(2*M^2*X1*X*F^4-
(2*M^2*X1*X*F^3*2*v)+(2*M^2*X1*X*F^2*v^2))/(C*(R1*R)^2)+
(M^2*X^2*F^4-(M^2*X^2*F^3*2*v)+(M^2*X^2*F^2*v^2))/(C*(R1*R)^2)+
(2*M*X1*X^2*F^4-
(2*M*X1*X^2*F^3*2*v)+(2*M*X1*X^2*F^2*v^2))/(C*(R1*R)^2)+
(X1^2*X^2*F^4-(X1^2*X^2*F^3*2*v)+(X1^2*X^2*F^2*v^2))/(C*(R1*R)^2)+
(2*M*X1^2*X*F^4-
(2*M*X1^2*X*F^3*2*v)+(2*M*X1^2*X*F^2*v^2))/(C*(R1*R)^2)-
(X2*X^2*F^4)/((R*C)^2)-(X2*M^2*F^4-
(X2*M^2*F^3*2*v)+(X2*M^2*F^2*v^2))/((R1*C)^2)-
(2*X2*M*X1*F^4-(2*X2*M*X1*F^3*2*v)+(2*X2*M*X1*F^2*v^2))/((R1*C)^2)-
(X2*X1^2*F^4-(X2*X1^2*F^3*2*v)+(X2*X1^2*F^2*v^2))/((R1*C)^2)-
(2*X2*M^2*F^4-(2*X2*M^2*F^3*2*v)+(2*X2*M^2*F^2*v^2))/(C*(R1*R)^2)-
(4*X2*M*X1*X*F^4-
(4*X2*M*X1*X*F^3*2*v)+(4*X2*M*X1*X*F^2*v^2))/(C*(R1*R)^2)-
(2*X2*X1^2*X*F^4-
(2*X2*X1^2*X*F^3*2*v)+(2*X2*X1^2*X*F^2*v^2))/(C*(R1*R)^2)-
(M*X^2*F^4)/((R*C)^2)+
(M*F^2)/((C)^2)+
(X2*F^2)/((C)^2)+
(2*M*X*F^2)/(C*(R)^2)+
(X2*M^2*F^2-(X2*M^2*2*v*F)+(X2*M^2*v^2))/((R1*R)^2)+
(2*X2*M*X1*F^2-(2*X2*M*X1*2*v*F)+(2*X2*M*X1*v^2))/((R1*R)^2)+
(X2*X1^2*F^2-(X2*X1^2*2*v*F)+(X2*X1^2*v^2))/((R1*R)^2)+
```

```
(2*X2*X*F^2)/(C*(R)^2)+
(M^2*F^2-(M^2*2*v*F)+(M^2*v^2))/(C*(R1)^2)+
(2*M*X1*F^2-(2*M*X1*2*v*F)+(2*M*X1*v^2))/(C*(R1)^2)+
(X1^2*F^2-(X1^2*2*v*F)+(X1^2*v^2))/(C*(R1)^2)-
(X^2*F^2)/(C*(R)^2)-
(M^2*X1*F^2-(M^2*X1*2*v*F)+(M^2*X1*v^2))/((R1*R)^2)-
(M*X1^2*F^2-(M*X1^2*2*v*F)+(M*X1^2*v^2))/((R1*R)^2)-
(M^2*X*F^2-(M^2*X*2*v*F)+(M^2*X*v^2))/((R1*R)^2)-
(2*M*X1*X*F^2-(2*M*X1*X*2*v*F)+(2*M*X1*X*v^2))/((R1*R)^2)-
(2*X1^2*X*F^2-(2*X1^2*X*2*v*F)+(2*X1^2*X*v^2))/((R1*R)^2)+
(X)/((R)^2)-
1/((C)^2)-
(M)/((R)^2)-
(X2)/((R)^2)=0
```

For F, the polynomials are of order 6.
For C, the polynomials are of order 2.
For v, the polynomials are of order 2.

Appendix B

1.B Symbolic Solution For C in Terms of F & v

```
C1=[ 1/2/(X*X1*R*v^2-M^2*F*R1*v+X*X1*R*F^2-R2*R1^2-2*X*M^2*R*F*v-X*R*R1^2-
2*X*X1*R*F*v+2*M*X1*R2*v^2+X*M^2*R*v^2+2*X*M*X1*R*v^2+M^2*R2*v^2+X1^2*R2*v^2+
2*M*X1*R2*F^2-4*M*X1*R2*F*v-2*M*X1*F^2*R1+2*M*X1*F*R1*v-2*M^2*R2*F*v-
4*X*M*X1*R*F*v+X*M^2*R*F^2+2*X*M*X1*R*F^2-
2*X1^2*R2*F*v+M^2*F^2*R1+X1^2*R2*F^2+M^2*R2*F^2)*(-2*X*X1^2*F^3*R-
4*X*M*X1*F^3*R1+2*X*M^2*F^3*R2-
4*X*M*X1*F^3*R+2*X*X1*F^3*R2+4*X*M*X1*F^3*R2+4*F^2*X*M*X1*R1*v+2*F^2*X*M^2*R
1*v+8*F^2*X*M*X1*R*v-4*F^2*X*M^2*R2*v-8*F^2*X*M*X1*R2*v-
4*F^2*X*X1*R2*v+4*F^2*X*X1^2*R*v-
4*F*X*M*X1*R*v^2+4*F*X*M*X1*R2*v^2+2*F*X*X1*R2*v^2-2*F*X*R1^2*R2-
2*F*X*R*R1^2+2*F*X*M^2*R2*v^2-2*F*X*X1^2*R*v^2-2*F*X*M^2*R1*v^2-2*(-
6*X^3*X1*R*v^2*M^2*F^4*R2+R2^2*R1^4*R^2-12*X^3*X1^2*R*v^2*M*F^4*R2-
6*X^3*X1^2*R*F^5*M*R1*v-3*X*X1*R^3*F^3*M^2*R1*v-
6*X*X1^2*R^3*F^3*M*R1*v+8*X^3*X1^2*R*F^5*M*R2*v+12*M^3*F^2*R1*v^2*X1*R2*R^2-
12*M^3*F^3*R1*v*X1*R2*R^2-6*M^2*F^3*R1*v*X1^2*R2*R^2-6*M^4*F^3*R1*v*R2*R^2-
8*X*X1^2*R^3*v^3*M*F*R2-2*X^3*X1^2*R*v^3*M*F^3*R1+6*X^3*X1^2*R*v^2*M*F^4*R1-
X*X1*R^3*v^3*M^2*F*R1-2*X*X1^2*R^3*v^3*M*F*R1-
2*X^3*X1^2*R*v^4*M*F^2*R2+8*X^3*X1^2*R*v^3*M*F^3*R2+M^4*F^2*R1^2*v^2*R^2+6*X*X1
*R^3*v^2*M^2*F^2*R2+12*X*X1^2*R^3*v^2*M*F^2*R2+6*X^2*X1^4*F^4*R^2*v^2+2*X^2*X1^2
*F^4*R^2*R1^2+3*X*X1*R^3*F^2*v^2*M^2*R1-
4*X^2*X1^4*F^5*R^2*v+6*X*X1^2*R^3*F^2*v^2*M*R1-2*X^2*X1^3*F^6*R*R2-
4*X*X1*R^3*F^3*v*M^2*R2-24*M*X1^3*R2^2*v^2*X^2*F^4+4*X^3*X1*R*F^5*v*M^2*R2-
4*X*X1^3*R^3*F^3*v*R2+2*X*X1*R^3*F*v*R2*R1^2+2*X^3*R*R1^3*M*X1*F^3*v+2*X*R^3*R1
^3*M*X1*F*v+6*M^4*R2^2*v^2*F^2*R^2-4*X*R^3*R1^2*M*X1*F^2*R2-
2*X*R^3*R1^2*M^2*F^2*R2+4*X^3*R*R1^2*M*X1*F^4*R2+24*X*M^3*R^3*F^2*v^2*X1*R2+X^
3*R*R1^2*X1^2*F^4*R2+2*X^3*R*R1^2*M^2*F^4*R2+12*X*M^3*R^3*F^2*v^2*X1*R1+16*X^3*
M^3*R*F^3*v^3*X1*R2-24*X^3*M^3*R*F^4*v^2*X1*R2+X*R^3*R1^4*R2-
16*X*M^3*R^3*F^3*v*X1*R2-
20*X*M^2*R^3*F^3*v*X1^2*R2+20*X^3*M^2*R*F^5*v*X1^2*R2+16*X^3*M^3*R*F^5*v*X1*R2+
4*X*M^2*R^3*F*v*R2*R1^2+8*R2^2*R1^2*M*X1*F*R^2*v-8*X*X1^2*R^3*F^3*M*R2*v-
16*X*M^3*R^3*v^3*X1*F*R2-2*X^2*X1*F^4*R2^2*R1^2-4*X*M^3*R^3*v^3*X1*F*R1-
4*X^3*M^3*R*v^4*X1*F^2*R2+30*X*M^2*R^3*v^2*X1^2*F^2*R2+6*X^2*X1^2*F^4*R2^2*v^2-
4*X^2*X1^2*F^5*R2^2*v-6*X^3*M^4*R*v^2*F^4*R2-
24*M^2*X1^2*R2^2*v^3*F*R^2+4*X^2*X1^2*F^6*R2^2*M-
4*M^3*X1*R2*v^3*F*R1*R^2+4*X^2*M^2*X1^2*F^6*R^2-
2*M^2*X1^2*R2*v^3*F*R1*R^2+2*X^2*M^2*F^6*R2^2*X1-4*M*X1*R2^2*v^2*R1^2*R^2-
4*X*M^2*X1^2*R^3*v^3*F*R1-
5*X^3*M^2*X1^2*R*v^4*F^2*R2+20*X^3*M^2*X1^2*R*v^3*F^3*R2-2*M^2*R2^2*v^2*R1^2*R^2-
2*F^3*X^2*M^4*R1^2*v^3+12*X*M*X1^3*R^3*v^2*F^2*R2-30*X^3*M^2*R*v^2*X1^2*F^4*R2-
4*X*M*X1*R^3*v^2*R2*R1^2-2*X^2*M^2*X1^2*F^6*R2^2-
3*X^3*X1*R*v^2*M^2*F^4*R1+2*X*X1^2*R^3*v^4*M*R2-12*X^3*M*X1^3*R*v^2*F^4*R2-
X^3*X1*R*v^4*M^2*F^2*R2+F^2*X^2*X1^2*R2^2*v^4+X^3*X1*R*v^2*F^2*R1^2*R2-
4*X*X1*R^3*v^3*M^2*F*R2-4*X1^3*R2*v^3*X^2*M*F^3*R1+12*X1^3*R2*v^2*X^2*M*F^4*R1-
4*F^3*X^2*X1^4*R^2*v^3-16*X1^3*R2^2*v^3*M*F*R^2+X*X1^3*R^3*v^4*R2-
2*X1^2*R2^2*v^2*R1^2*R^2-20*X*M^2*X1^2*R^3*v^3*F*R2-
4*F^3*X^2*X1^2*R2^2*v^3+12*X^3*M^2*X1^2*R*v^2*F^4*R1-
4*X^3*M^2*X1^2*R*v^3*F^3*R1+6*M^2*F^2*R1*v^2*X1^2*R2*R^2+X^3*X1*R*v^3*M^2*F^3*R
1+2*F^2*X^2*R1^4*R2*R+4*X^3*X1*R*v^3*M^2*F^3*R2-
10*X^2*X1^2*F^6*R*M^2*R2+F^2*X^2*X1^4*R^2*v^4+X^3*R*R1^2*X1^2*F^2*R2*v^2-
2*X^3*R*R1^2*X1^2*F^3*R2*v-
2*X*R^3*R1^3*M*X1*F^2+2*X^3*R*R1^2*M^2*F^2*R2*v^2+2*X*R^3*R1^2*X1^2*F*R2*v-
X^3*R*R1^3*M^2*F^3*v-2*R2^2*R1^2*M^2*F^2*R^2-
2*R2^2*R1^2*X1^2*F^2*R^2+X^2*X1^2*F^6*R2^2-
26*X^2*X1^2*F^5*R*M^2*R1*v+X^2*X1^4*F^6*R^2+2*X^2*X1^2*F^4*R*R1^2*R2+4*X^2*X1^3*
```

```
F^6*R*M*R1-12*X^2*X1^3*F^4*R*R2*v^2+2*R2^2*R1^2*X^2*X1^2*F^4-
12*X*M^3*R^3*F^3*v*X1*R1-
4*X^3*M^2*R*F^3*v*R1^2*R2+4*X^3*M^4*R*F^3*v^3*R2+X*X1^3*R^3*F^4*R2+3*X^3*X1*R*F
^5*M^2*R1*v-
12*X^2*X1^3*F^5*R*M*R1*v+4*X^2*X1^3*F^6*R^2*M+24*X^2*X1^3*F^4*R^2*M*v^2+8*X^2*X
1^3*F^5*R*R2*v+2*R2*R1^3*X^2*M^2*F^4-X^3*X1^3*R*F^6*R2-16*X^2*X1^3*F^5*R^2*M*v-
4*X^2*X1^3*F^6*R*M*R2-60*X^2*X1^2*F^4*R*M^2*R2*v^2-2*R2*R1^3*M^2*F^2*R^2-
8*X^2*M^2*F^5*R2^2*X1*v-2*M^4*F^3*R1^2*v*R^2-
24*X^2*X1^3*F^4*R*M*R2*v^2+2*M^4*F^5*R1^2*v*X^2+16*X^2*X1^3*F^5*R*M*R2*v+F^2*X^
2*R^2*R1^4+6*X1^4*R2^2*v^2*F^2*R^2+40*X^2*X1^2*F^5*R*M^2*R2*v+4*M^3*X1*R2^2*v^4*
R^2+8*X^2*M^4*F^5*R2*R1*v-
4*X^2*M^3*F^6*R2*X1*R+12*X^2*M^3*X1*F^4*R1^2*v^2+4*X^2*M*X1*F^4*R1^3*R-
6*X1^4*R2^2*v^2*X^2*F^4-2*X^3*X1*R*F^3*v*R1^2*R2+4*M*X1^3*R2^2*v^4*R^2-
12*X^2*M^3*X1*F^5*R1^2*v-2*X^2*M^2*X1^2*F^6*R1*R2-
4*X^2*M*X1^2*F^6*R1*R2+8*X^2*M^2*X1^2*F^6*R1*R-4*X^2*M^3*X1*F^6*R1*R2-
4*X^2*M*X1^2*F^6*R*R2+30*X^2*X1^2*F^4*R*M^2*R1*v^2+6*M^2*X1^2*R2^2*v^4*R^2-
12*X^2*M^4*F^4*R2*R1*v^2-
2*X^2*M^2*F^4*R2*R*R1^2+12*X^2*M^2*F^4*R2^2*X1*v^2+2*M^2*X1^2*F^4*R1*R2*R^2-
2*X^2*X1*F^4*R2*R*R1^2+24*X^2*X1^2*F^4*R2^2*M*v^2+4*M*X1^3*F^6*R1*X^2*R2+12*X^2
*M*X1^2*F^5*R1*R2*v+12*X*M^2*X1^2*R^3*v^2*F^2*R1+X1^4*R2^2*v^4*R^2-
16*X^2*X1^2*F^5*R2^2*M*v-2*X^3*M*X1^3*R*v^4*F^2*R2-
X1^4*R2^2*v^4*X^2*F^2+6*X^2*M^2*X1^2*F^5*R1*R2*v+4*X1^4*R2^2*v^3*X^2*F^3+8*X^3*M
*X1^3*R*v^3*F^3*R2-4*M^4*R2^2*v^3*F*R^2-
4*X1^4*R2^2*v^3*F*R^2+4*X^2*M*X1*F^4*R^2*R1^2-
X*R^3*R1^3*M^2*F^2+24*X^2*M^2*X1^2*F^4*R^2*v^2+16*X^2*M^3*X1*F^5*R1*R2*v+X^3*R*
R1^3*M^2*F^4-8*X*M*X1^3*R^3*v^3*F*R2+M^4*R2^2*v^4*R^2-
X^3*R*R1^4*F^2*R2+X*M^4*R^3*v^4*R2-16*X^2*M^2*X1^2*F^5*R^2*v-
X^3*M^4*R*v^4*F^2*R2+4*M^3*X1*R2^2*F^4*R^2-12*M*X1^3*F^5*R1*v*X^2*R2-
4*M^2*X1^2*F^2*R1^2*v^2*R^2+2*M^4*F^4*R1*R2*R^2+6*M^2*X1^2*R2^2*F^4*R^2-
12*X^2*M^2*X1^2*F^4*R2^2*v^2-
24*X^2*M^3*X1*F^4*R1*R2*v^2+4*M*X1^3*R2^2*F^4*R^2+8*X^2*M^2*X1^2*F^5*R2^2*v-
12*X^2*M*X1^2*F^4*R1*R2*v^2-
6*X^2*M^2*X1^2*F^4*R1*R2*v^2+8*M^2*X1^2*F^3*R1^2*R^2*v-
4*M*X1^3*R2^2*F^6*X^2+4*M^3*X1*F^4*R1*R2*R^2+8*X^3*M*X1^3*R*F^5*v*R2-
4*X^3*M^3*R*F^6*X1*R2+M^4*R2^2*F^4*R^2-
4*X1^4*R2^2*F^3*v*R^2+16*X^2*M^3*F^5*R2*X1*R*v-
2*M^4*F^6*R1*X^2*R2+12*X^2*M*X1^3*F^4*R1*R*v^2-
4*M^4*R2^2*F^3*v*R^2+4*M^3*F^6*R1^2*X^2*X1-4*X^2*M^3*X1*F^5*R*R1*v-
16*F^3*X^2*M*X1^3*R^2*v^3-16*F^3*X^2*M^2*X1^2*R^2*v^3-
24*X^2*M^3*F^4*R2*X1*R*v^2+4*X^3*M^2*X1^2*R*F^6*R1-
12*X*M^2*X1^2*R^3*F^3*R1*v+8*F^3*X^2*M^4*R1*v^3*R2-2*F^3*X^2*M^2*R1^3*v*R-
4*F^3*X^2*M^2*R1^3*v*R2+2*X*M*X1^3*R^3*F^4*R2-2*X^3*M*X1^3*R*F^6*R2-
4*F^3*X^2*M^3*X1*R1^2*v^3+4*X*M^3*R^3*F^4*X1*R2+X*M^4*R^3*F^4*R2+5*X*M^2*R^3*F
^4*X1^2*R2-5*X^3*M^2*R*F^6*X1^2*R2-12*X^3*M^2*X1^2*R*F^5*v*R1-
X^3*M^4*R*F^6*R2+4*X1^4*R2^2*F^5*v*X^2-
4*M^2*X1^2*F^4*R1^2*R^2+8*F^3*X^2*X1^3*R2*v^3*R+4*F^3*X^2*X1*R2^2*v*R1^2-
8*X*M*X1^3*R^3*F^3*v*R2-6*X^2*X1*F^4*R2*M^2*R1*v^2+X*M^4*R^3*F^4*R1-
X^3*M^4*R*F^6*R1-
16*F^3*X^2*M*X1^2*R2^2*v^3+2*X^2*X1*F^5*R2*M^2*R1*v+8*F^3*X^2*M^2*X1^2*R2^2*v^3+
12*X^2*M^3*X1*F^4*R*R1*v^2-24*X^2*M*X1^2*F^4*R*R2*v^2-
X1^4*R2^2*F^6*X^2+X1^4*R2^2*F^4*R^2-
8*F^3*X^2*M^2*R2^2*v^3*X1+16*X^2*M*X1^2*F^5*R*R2*v+M^4*F^4*R1^2*R^2+4*X*M^3*R^3
*F^4*X1*R1+4*F^2*X^2*M*X1^3*R^2*v^4+4*F^2*X^2*M^2*X1^2*R^2*v^4-
4*F^3*X^2*X1^2*R^2*v*R1^2-2*F^2*X^2*X1*R2^2*v^2*R1^2-
M^4*F^6*R1^2*X^2+4*X*M^2*X1^2*R^3*F^4*R1-
4*F^3*X^2*M*X1^3*R1*v^3*R+16*F^3*X^2*M^3*X1*R1*v^3*R2-
4*F^3*X^2*M*X1*R1^3*v*R+4*F^2*X^2*M*X1^2*R2^2*v^4-
2*F^2*X^2*M^2*X1^2*R2^2*v^4+4*F^3*X^2*M*X1^2*R1*v^3*R2+2*F^3*X^2*M^2*X1^2*R1*v^3
*R2-14*F^3*X^2*M^2*X1^2*R1*v^3*R+6*F^3*X^2*M^2*R1*v^3*X1*R2-
12*F^3*X^2*M^3*R1*v^3*X1*R-
```

2*F^2*X^2*X1^3*R2*v^4*R+2*F^2*X^2*X1*R2^2*v^4*M^2+2*F^2*X^2*R1^3*R2*M^2*v^2+2*F^2*X^2*R*R1^3*M^2*v^2+2*F^2*X^2*R^2*R1^2*X1^2*v^2-
2*F^2*X^2*M^4*R2*v^4*R1+16*F^3*X^2*M^3*X1*R*v^3*R2-
8*F^3*X^2*M*X1*R^2*v*R1^2+16*F^3*X^2*M*X1^2*R*v^3*R2+40*F^3*X^2*M^2*X1^2*R*v^3*R2+F^2*X^2*M^4*R1^2*v^4-6*X^3*X1^3*R*v^2*F^4*R2-
X*X1*R^3*v^2*R2*R1^2+4*F^3*X^2*M^2*R2*v*R*R1^2+2*M^2*F*R1^3*v*R2*R^2+6*X*X1^3*R^3*v^2*F^2*R2-X*X1*R^3*F^2*R2*R1^2-
4*R2^2*R1^2*M*X1*F^2*R^2+2*X^3*X1^2*R*F^6*M*R1+X*X1*R^3*F^4*M^2*R2+2*X*X1^2*R^3*F^4*M*R2-X^3*X1*R*F^6*M^2*R2-2*X^3*X1^2*R*F^6*M*R2+3*X*M^4*R^3*F^2*v^2*R1-
4*X*M^4*R^3*F^3*v*R2+4*X^3*M^4*R*F^5*v*R2+4*X^3*R*R1^2*M*X1*F^2*R2*v^2-
8*X^3*R*R1^2*M*X1*F^3*R2*v-2*X^3*R*R1^3*M*X1*F^4+X*R^3*R1^3*M^2*F*v-
X*R^3*R1^2*X1^2*F^2*R2+8*X*R^3*R1^2*M*X1*F*R2*v+4*X^3*X1^3*R*F^5*v*R2+24*M*X1^3*R2^2*v^2*F^2*R^2+24*M^3*X1*R2^2*v^2*F^2*R^2+36*M^2*X1^2*R2^2*v^2*F^2*R^2+6*X*M^4*R^3*v^2*F^2*R2+16*F^3*X^2*M*X1^3*R2*v^3*R-
2*X*M^2*R^3*v^2*R2*R1^2+4*F^2*X^2*M*X1*R^2*v^2*R1^2-4*F^2*X^2*M*X1^2*R*v^4*R2-
10*F^2*X^2*M^2*X1^2*R*v^4*R2-4*F^3*X^2*X1^2*R*v*R1^2*R2-
X*M^4*R^3*v^3*F*R1+4*F^3*X^2*X1*R2*v*R*R1^2-2*F^2*X^2*X1*R2*v^2*R*R1^2-
16*M^3*R2^2*v^3*X1*F*R^2-4*F^2*X^2*M^3*X1*R2*v^4*R1-4*F^2*X^2*M*X1^3*R2*v^4*R-
2*M^4*R2*v^3*F*R1*R^2+4*F^2*X^2*M^3*X1*R*v^4*R1-4*F^2*X^2*M^3*X1*R*v^4*R2-
2*F^2*X^2*X1*R2*v^4*M^2*R1+16*X1^3*R2^2*v^3*X^2*M*F^3+4*X^3*X1^3*R*v^3*F^3*R2-
4*X*X1^3*R^3*v^3*F*R2-
4*X1^3*R2^2*v^4*X^2*M*F^2+2*F^2*X^2*R1^2*R2*X1^2*R*v^2+X*X1*R^3*v^4*M^2*R2-
X^3*X1^3*R*v^4*F^2*R2+6*M^4*F^2*R1*v^2*R2*R^2+X^3*X1*R*F^4*R1^2*R2+4*R2^2*R1^2*X1^2*F*R^2*v+4*R2^2*R1^2*M^2*F*R^2*v+X*X1*R^3*F^4*M^2*R1+2*X*X1^2*R^3*F^4*M*R1-
X^3*X1*R*F^6*M^2*R1+2*R2^2*R1^2*X^2*X1^2*F^2*v^2-4*R2^2*R1^2*X^2*X1^2*F^3*v-
X*R^3*R1^2*X1^2*R2*v^2-2*F^2*X^2*R*R1^2*M^2*R2*v^2-4*X*M^4*R^3*F*v^3*R2-
3*X*M^4*R^3*F^3*v*R1-
3*X^3*M^4*R*F^4*v^2*R1+3*X^3*M^4*R*F^5*v*R1+2*X*M*X1^3*R^3*v^4*R2+5*X*M^2*R^3*v^4*X1^2*R2+4*X*M^3*R^3*v^4*X1*R2+X^3*M^4*R*v^3*F^3*R1-
24*M^2*X1^2*R2^2*F^3*R^2*v-
16*M*X1^3*R2^2*F^3*v*R^2+16*M*X1^3*R2^2*F^5*v*X^2+2*F^2*X^2*X1^2*R*v^4*M^2*R1-
16*M^3*X1*R2^2*F^3*v*R^2)^(1/2))*F]

C2=[1/2/(X*R1^2-M^2*X1*v^2-2*X1^2*X*v^2-M*X1^2*v^2+2*X2*M*X1*v^2-2*X2*M^2*F*v-
2*M*X1*X*F^2+2*M*X1^2*F*v+4*X1^2*X*F*v+2*X2*M*X1*F^2+4*M*X1*X*F*v-
4*X2*M*X1*F*v+2*M^2*X*F*v-2*X2*X1^2*F*v+2*M^2*X1*F*v-M*R1^2-X2*R1^2-
2*M*X1*X*v^2-M^2*X*v^2+X2*X1^2*v^2-M^2*X1*F^2-M*X1^2*F^2+X2*X1^2*F^2-
2*X1^2*X*F^2-M^2*X*F^2+X2*M^2*F^2+X2*M^2*v^2)*(-X1^2*F^2*R^2-M^2*F^2*R^2-
M^2*X^2*F^4-X1^2*X^2*F^4+2*M^2*R^2*F*v+2*X1^2*R^2*F*v-
2*M*X1*X^2*F^2*v^2+2*X2*M^2*F^4-2*M*X*F^2*R1^2-
8*X2*M*X1*X*F^3*v+4*M*X1*X^2*F^3*v-2*M^2*X1*X*F^4+4*X2*M*X1*X*F^2*v^2-
2*M^2*X1*X*F^2*v^2+4*M^2*X1*X*F^3*v-
4*X2*X1^2*X*F^3*v+2*M^2*X^2*F^3*v+4*M*X1^2*X*F^3*v-
M^2*X^2*F^2*v^2+2*X2*X1^2*X*F^2*v^2-X1^2*X^2*F^2*v^2+2*X1^2*X^2*F^3*v-
2*M*X1^2*X*F^4+2*X2*X1^2*X*F^4-2*M*X1*R^2*v^2-
2*M*X1^2*X*F^2*v^2+4*M*X1*R^2*F*v-2*X2*X*F^2*R1^2+4*X2*M*X1*X*F^4-
X1^2*R^2*v^2+X^2*F^2*R1^2+2*X2*M^2*F^2*v^2-4*X2*M^2*F^3*v-M^2*R^2*v^2-
2*M*X1*F^2*R^2-2*M*X1*X^2*F^4-(-2*X1^2*X^2*F^4*R1^2*R^2-2*M^2*X^2*F^4*R1^2*R^2-
4*M*X*F^2*R1^4*R^2+M^4*F^4*R^4+M^4*X^4*F^8+X1^4*X^4*F^8+12*M^2*X1*X*F^2*v^2*R1^2*R^2-16*M*X1^2*X*F^3*v*R1^2*R^2-32*X2*X1^2*X*F^3*v*R1^2*R^2-
24*M^2*X1*X*F^3*v*R1^2*R^2+16*X2*X1^2*X*F^2*v^2*R1^2*R^2-
2*X1^2*X^2*F^2*v^2*R1^2*R^2+8*M*X1^2*X*F^4*R1^2*R^2+2*M^4*F^6*R^2*X^2-
4*X1^4*F^3*R^4*v+2*X1^4*F^6*R^2*X^2+6*X1^2*F^4*R^4*M^2+4*X1^2*X^2*F^3*v*R1^2*R^2+6*X1^4*R^4*F^2*v^2+6*M^4*R^4*F^2*v^2-
4*X1^3*X^3*F^8*M^2+16*X2*X1^2*X*F^4*R1^2*R^2-4*M^4*X^4*F^7*v-
4*M^4*X^2*F^8*X2+6*M^2*X^4*F^8*X1^2-4*M^4*F^6*R^2*X2-4*M^4*F^3*R^4*v-
48*X2*M*X1*X*F^3*v*R1^2*R^2-4*X1^4*R^4*F*v^3-
4*M^4*R^4*F*v^3+4*X1^3*X^4*F^8*M+4*X1^4*X^3*F^8*X2-4*X1^4*X^3*F^8*M-
4*X1^4*X^4*F^7*v+6*X1^4*X^4*F^6*v^2+4*M^3*X^4*F^8*X1-

2*M^2*X^4*F^6*R1^2+4*M^3*F^4*R^4*X1+4*X1^3*F^4*R^4*M+24*X2^2*M^4*F^6*v^2+12*M^2
*X1*X*F^4*R1^2*R^2-
4*X2*X*F^2*R1^4*R^2+4*M^3*X1*R^4*v^4+4*M*X1^3*R^4*v^4+6*M^2*X1^2*R^4*v^4-
4*M*X1*X^2*F^2*v^2*R1^2*R^2-
4*X1^4*X^4*F^5*v^3+X1^4*X^4*F^4*v^4+M^4*X^4*F^4*v^4+8*M*X1^2*X*F^2*v^2*R1^2*R^2-
16*X2^2*M^4*F^7*v-16*X2^2*M^4*F^5*v^3+X1^4*F^4*R^4-
4*M*X1*X^2*F^4*R1^2*R^2+4*X2^2*M^4*F^4*v^4+24*X2*M*X1*X*F^4*R1^2*R^2-
8*X2^2*M^2*F^6*X*R1^2+16*X2^2*M^3*F^8*X1*X+4*X2*M^2*F^6*X^2*R1^2-
8*X2*M^3*F^6*X1*R^2-
8*X2*M^3*F^8*X1^2*X+8*X2^2*M^2*F^8*X1^2*X+48*X2^2*M^2*F^6*X1^2*X*v^2-
4*M*X1*X^4*F^4*v^2*R1^2-
8*M^3*X1*X^2*F^4*v^4*X2+32*M^3*X1*X^2*F^5*v^3*X2+8*M^3*X1*X^2*F^2*v^4*R^2+8*M*X
1^3*X^2*F^2*v^4*R^2-48*X2*M^3*F^4*X1*R^2*v^2-
48*X2*M^3*F^6*X1^2*X*v^2+32*X2*M^3*F^5*X1*R^2*v+4*M^3*X1*X^4*F^4*v^4+8*M*X1^3*
X^3*F^4*v^4*X2+4*M*X1^3*X^4*F^4*v^4-
16*M*X1^3*X^4*F^5*v^3+16*X1^2*R^2*F^3*v^3*X2*M^2-
48*X1^4*R^2*F^3*v^3*M*X+12*M^2*X1^2*X^2*F^2*v^4*R^2-4*M^2*X1^3*X^3*F^4*v^4-
24*M^2*R^2*F^3*v*X2*X*R1^2-
24*M^2*R^4*F*v^3*X1^2+16*M^4*R^2*F^3*v^3*X2+48*X1^4*R^2*F^3*v^3*X2*X-
8*X1^4*R^2*F^3*v^3*X^2-16*X1^3*R^4*F*v^3*M+192*M^2*R^2*F^3*v^3*X2*X1^2*X-
48*M^2*R^2*F^3*v^3*X1^2*X^2-16*M^3*R^4*F*v^3*X1-
96*M^3*R^2*F^3*v^3*X1^2*X+24*X1^4*X^3*F^6*X2*v^2-24*X1^4*X^3*F^6*M*v^2-
4*X1^2*X^3*F^6*X2*R1^2-2*X1^2*X^4*F^6*R1^2-
24*M^4*X^2*F^6*X2*v^2+8*M^3*X^3*F^8*X2*X1+8*X1^3*X^3*F^8*X2*M-
24*X1^2*X^2*F^6*X2*M^2*v^2+16*X1^2*X^2*F^7*X2*M^2*v-
8*M^4*R^2*F^3*v^3*X^2+8*M^3*F^6*R^2*X1*X^2+8*M^2*X^3*F^8*X2*X1^2-
4*M^2*X^3*F^6*X2*R1^2-48*M^2*F^6*R^2*X2*X1^2*X+24*M^3*F^2*R^4*X1*v^2-
16*M^3*F^3*R^4*X1*v+12*M^2*F^4*R^2*X2*X*R1^2-24*M^3*F^6*R^2*X2*X1*X-
24*M^4*F^4*R^2*X2*v^2-
24*X1^2*F^4*R^2*X2*M^2*v^2+8*X1^3*F^6*R^2*M*X^2+24*M^3*F^6*R^2*X1^2*X-
12*X1^4*F^6*R^2*X2*X+24*X1^3*F^2*R^4*M*v^2-16*X1^3*F^3*R^4*M*v-
40*X1^3*F^6*R^2*X2*M*X+12*X1^4*F^4*R^2*X^2*v^2+12*X1^4*F^6*R^2*M*X-
8*X2^2*M^2*X1^2*X^2*F^4*v^4+8*X2*M^3*X1^2*X^2*F^4*v^4+6*M^4*X^4*F^6*v^2-
32*X2*M^3*X1*X^3*F^5*v^3+36*M^2*X1^2*X^4*F^6*v^2-
32*X2*M^3*X1^2*X^2*F^5*v^3+48*X2*M^3*X1^2*X^2*F^6*v^2-
48*X2^2*M^2*X1^2*X^2*F^6*v^2+48*X2*M^2*X1^2*X^3*F^6*v^2-
32*X2*M^3*X1^2*X^2*F^7*v+32*X2^2*M^2*X1^2*X^2*F^5*v^3+32*X2*M^4*F^7*X1*X*v-
32*X2^2*M^2*F^7*X1^2*X*v+16*X2*M^4*F^7*X^2*v-
64*X2^2*M^3*F^7*X1*X*v+32*X2*M^3*F^7*X1*X^2*v-
8*X2*M^4*F^8*X1*X+96*X2^2*M^3*F^6*X1*X*v^2+16*M^2*X1^3*X^3*F^5*v^3-
8*X2*M^3*F^6*X*R1^2-48*X2*M^4*F^6*X1*X*v^2+32*X2*M^3*F^7*X1^2*X*v-
32*M^2*X1^2*X^3*F^5*v^3*X2-
24*M^2*X1^2*X^4*F^5*v^3+8*M^2*X1^2*X^3*F^4*v^4*X2+6*M^2*X1^2*X^4*F^4*v^4-
48*M^3*X1*X^2*F^6*v^2*X2+8*M*X1*X^2*F^3*v*R1^2*R^2-112*X1^3*R^2*F^3*v^3*M^2*X-
72*X1^4*R^2*F^4*v^2*X2*X+72*X1^2*R^2*F^4*v^2*M^2*X^2+160*X1^3*R^2*F^3*v^3*X2*M*X
-
32*X1^3*R^2*F^3*v^3*M*X^2+16*X1^2*R^2*F^5*v*X2*M^2+144*M^3*R^2*F^4*v^2*X1^2*X+72
*X1^4*R^2*F^4*v^2*M*X-32*M*X1^3*X^3*F^5*v^3*X2-16*M^3*X1*X^4*F^5*v^3-
32*M^4*R^2*F^3*v^3*X1*X-
288*M^2*R^2*F^4*v^2*X2*X1^2*X+12*M^4*R^2*F^4*v^2*X^2+4*X2^2*M^4*F^8+96*M^3*R^2*F
^3*v^3*X2*X1*X+36*M^2*R^4*F^2*v^2*X1^2-
32*M^3*R^2*F^3*v^3*X1*X^2+16*M^4*R^2*F^5*v*X2-
16*M^3*R^2*F^3*v*X*R1^2+16*X1^4*X^3*F^7*M*v+16*X1^3*X^3*F^7*M^2*v-
16*X1^4*X^3*F^7*X2*v-24*X1^2*X^4*F^7*M^2*v-4*X1^2*X^2*F^8*X2*M^2-
4*X1^2*X^3*F^6*M*R1^2-16*X1^3*X^4*F^7*M*v-
8*X1^4*X^2*F^5*R^2*v+24*X1^3*X^4*F^6*M*v^2-
32*X1^3*X^3*F^7*X2*M*v+48*X1^3*X^3*F^6*X2*M*v^2-
24*X1^3*X^3*F^6*M^2*v^2+24*M^3*X^4*F^6*X1*v^2-32*M^3*X^3*F^7*X2*X1*v-
16*M^3*X^4*F^7*X1*v+48*M^3*X^3*F^6*X2*X1*v^2-
32*M^2*X^3*F^7*X2*X1^2*v+48*M^3*F^4*R^2*X1*X^2*v^2+8*M^3*F^4*R^2*X*R1^2+96*M^3*

F^5*R^2*X2*X1*X*v-32*M^3*F^5*R^2*X1*X^2*v-
112*X1^3*F^5*R^2*M^2*X*v+48*X1^4*F^5*R^2*X2*X*v-48*X1^2*F^5*R^2*M^2*X^2*v-
48*X1^4*F^5*R^2*M*X*v+168*X1^3*F^4*R^2*M^2*X*v^2+8*M^4*F^6*R^2*X1*X-
144*M^3*F^4*R^2*X2*X1*X*v^2+48*M^4*F^4*R^2*X1*X*v^2-
32*M^4*F^5*R^2*X1*X*v+192*M^2*F^5*R^2*X2*X1^2*X*v-8*M^4*F^5*R^2*X^2*v-
96*M^3*F^5*R^2*X1^2*X*v-32*X1^3*F^5*R^2*M*X^2*v+28*X1^3*F^6*R^2*M^2*X-
240*X1^3*F^4*R^2*X2*M*X*v^2-24*X1^2*F^3*R^4*M^2*v+48*X1^3*F^4*R^2*M*X^2*v^2-
4*X1^2*F^6*R^2*X2*M^2+160*X1^3*F^5*R^2*X2*M*X*v+12*X1^2*F^6*R^2*M^2*X^2-
4*X2*M^4*F^2*v^4*R^2-4*X1^2*R^2*v^4*X2*M^2*F^2+X^4*F^4*R1^4+X1^4*R^4*v^4-
8*X2^2*M^2*X1^2*X^2*F^8-
8*X2^2*X*F^4*R1^2*M^2*v^2+16*X2^2*X*F^5*R1^2*M^2*v+12*X2*X*F^2*R1^2*M^2*R^2*v^2-
8*M^3*X1^2*X*F^4*v^4*X2-12*X2*X1^4*X*F^2*v^4*R^2+12*M*X1^4*X*F^2*v^4*R^2-
8*M^3*X1*R^2*v^4*X2*F^2+32*M^3*X1*R^2*v^3*X2*F^3+4*X1^2*X^4*F^5*v*R1^2+2*X1^4*X^
2*F^2*v^4*R^2-2*X1^2*X^4*F^4*v^2*R1^2-
4*X1^2*X^2*F^4*v^4*X2*M^2+16*X1^2*X^2*F^5*v^3*X2*M^2-4*X1^4*X^3*F^4*v^4*M-
4*X2*X1^2*X^3*F^4*v^2*R1^2+8*X2^2*X1^2*X*F^4*v^4*M^2+2*M^4*X^2*F^2*v^4*R^2-
4*M^2*X^3*F^4*v^2*X2*R1^2-2*M^2*X^4*F^4*v^2*R1^2-
4*M^4*X^2*F^4*v^4*X2+4*X2*X1^4*X^3*F^4*v^4+32*M^3*X1^2*X*F^5*v^3*X2+4*M^2*X^4*F^
5*v*R1^2+16*M^4*X^2*F^5*v^3*X2+16*M*X1^4*X^3*F^5*v^3+8*M^2*X^3*F^5*v*X2*R1^2+8*X
2*X1^2*X^3*F^5*v*R1^2-32*X2^2*X1^2*X*F^5*v^3*M^2-
16*X2*X1^4*X^3*F^5*v^3+28*M^2*X1^3*X*F^2*v^4*R^2-4*M^4*X^4*F^5*v^3-
8*M^4*X1*X*F^4*v^4*X2+32*M^4*X1*X*F^5*v^3*X2+8*M^4*X1*X*F^2*v^4*R^2+24*M^3*X1^
2*X*F^2*v^4*R^2-40*X2*M*X1^3*X*F^2*v^4*R^2+16*X2^2*M^3*X1*X*F^4*v^4-
48*X2*M^2*X1^2*X*F^2*v^4*R^2-
24*X2*M^3*X1*X*F^2*v^4*R^2+8*M^3*X1^2*X^2*F^8*X2+8*X2*M^3*X1*X^3*F^4*v^4+8*M*X
1*X^4*F^5*v*R1^2+32*X2^2*M^2*X1^2*X^2*F^7*v-64*X2^2*M^3*X1*X*F^5*v^3-
8*M^3*X*F^4*R1^2*X2*v^2+16*M^3*X*F^5*R1^2*X2*v+8*M^3*X*F^2*R1^2*R^2*v^2-
8*X2*M^3*F^8*X1*X^2-4*M*X^3*F^4*R1^2*X1^2*v^2+8*M*X^3*F^5*R1^2*X1^2*v-
16*X2^2*M^4*F^3*v^3*R^2+32*X2*M^4*F^5*v*X1*R^2+96*X2*M^3*F^5*v*X1^2*R^2+32*M*X1
^4*F^5*v*X2*R^2+24*X2^2*M^4*F^4*v^2*R^2-
64*X2^2*M^3*F^5*v*X1*R^2+16*X2^2*M^3*X1*v^4*F^2*R^2-
96*X2^2*M^2*X1^2*v^3*F^3*R^2+24*X2^2*M^2*X1^2*v^4*F^2*R^2-
64*X2^2*M*X1^3*v^3*F^3*R^2+16*X2^2*M*X1^3*v^4*F^2*R^2-
16*M^2*X1^4*v^3*F^3*R^2+96*X2^2*M^3*X1*v^2*F^4*R^2+96*X2^2*M*X1^3*v^2*F^4*R^2-
64*X2^2*M^3*X1*v^3*F^3*R^2+24*M^2*X1^4*v^2*F^4*R^2+4*M^2*X1^4*v^4*F^2*R^2+144*X2^
2*M^2*X1^2*v^2*F^4*R^2-16*X2^2*M^4*F^5*v*R^2-
96*X2^2*M^2*F^5*v*X1^2*R^2+4*X*R1^4*R^2-
48*M*X1^4*v^2*X2*F^4*R^2+32*M*X1^4*v^3*X2*F^3*R^2-8*M*X1^4*v^4*X2*F^2*R^2-
32*M^3*X1^3*v^3*F^3*R^2+8*M^3*X1^3*v^4*F^2*R^2-
8*M^4*X1*v^4*X2*F^2*R^2+24*M^4*X1^2*v^2*F^4*R^2+48*M^3*X1^3*v^2*F^4*R^2-
16*M^4*X1^2*v^3*F^3*R^2+4*M^4*X1^2*v^4*F^2*R^2-
24*M^3*X1^2*v^4*X2*F^2*R^2+96*M^2*X1^3*v^3*X2*F^3*R^2-24*M^2*X1^3*v^4*X2*F^2*R^2-
48*M^4*X1*v^2*X2*F^4*R^2-144*M^2*X1^3*v^2*X2*F^4*R^2+32*M^4*X1*v^3*X2*F^3*R^2-
144*M^3*X1^2*v^2*X2*F^4*R^2+96*M^3*X1^2*v^3*X2*F^3*R^2+M^4*R^4*v^4+4*X^2*F^4*R1^
2*X2*M^2*v^2-8*X^2*F^5*R1^2*X2*M^2*v-4*X^4*F^6*R1^2*M*X1-
96*X2^2*M^3*X1*v^2*X^2*F^6+64*X2^2*M^3*X1*v^3*X^2*F^5-16*X2^2*M^3*X1*v^4*X^2*F^4-
8*X2*M^2*F*v*R1^2*R^2-16*X2*M^3*F^5*v*X^2*R1^2+16*X2*M^3*F^3*v*R1^2*R^2-
4*M*X1^2*v^2*R1^2*R^2+8*M^2*X1^2*v^2*F^2*R1^2*R^2-
8*X1^2*X*v^2*R1^2*R^2+8*M^4*X1*v^4*X2*X^2*F^4+48*M^4*X1*v^2*X2*X^2*F^6-
4*M^2*X1*v^2*R1^2*R^2+8*M^3*X1*v^2*F^2*R1^2*R^2-32*M^4*X1*v^3*X2*X^2*F^5-
8*M^2*X1*v^2*X2*F^2*R1^2*R^2-
24*M^4*X*F^4*v^2*X2*R^2+16*M^4*X*F^3*v^3*X2*R^2+16*M^4*X*F^5*v*X2*R^2-
24*X2*M^2*X1^3*F^6*R^2-64*X2^2*M*X1^3*F^5*R^2*v+24*X2^2*M^2*X1^2*F^6*R^2-
24*X2*M^3*X1^2*F^6*R^2+16*X2^2*M*X1^3*F^6*R^2+16*X2^2*M^3*X1*F^6*R^2-
16*M^2*X1^4*F^5*v*R^2-32*M^3*X1^3*F^5*v*R^2+96*M^2*X1^3*F^5*v*X2*R^2-
16*M^4*X^3*F^7*v*X2+24*M^4*X^3*F^6*v^2*X2-
16*X2*M*X1*F*v*R1^2*R^2+16*X2*M^2*X1*F^3*v*R1^2*R^2+32*X2^2*M*X1*F^3*v*R1^2*R^2
+16*M*X1*X*F*v*R1^2*R^2+4*M^4*X^3*F^8*X2-8*X2*M^2*X1*F^4*R1^2*R^2-
16*X2^2*M*X1*F^4*R1^2*R^2-
16*X2^2*M^3*X1*F^8*X^2+8*X2*M*X1*F^2*R1^2*R^2+16*X1^2*X*F*v*R1^2*R^2+8*M*X1^2*F

*v*R1^2*R^2-16*M^2*X1^2*F^3*v*R1^2*R^2-
8*M*X1*X*F^2*R1^2*R^2+16*X2^2*M^2*F^3*v*R1^2*R^2-
24*X2^2*M^4*F^6*v^2*X^2+16*X2^2*M^4*F^5*v^3*X^2+64*X2^2*M^3*F^7*v*X1*X^2-
16*X2^2*M^2*F^5*v*X^2*R1^2-32*X2*M^4*F^7*v*X1*X^2+16*X2^2*M^4*F^7*v*X^2-
16*X2^2*M*X1*v^2*F^2*R1^2*R^2+8*X2*M*X1*v^2*R1^2*R^2-
8*M*X1^4*F^6*X2*R^2+4*X2^2*M^4*F^6*R^2-
8*M^4*X1*F^6*X2*R^2+4*X2^2*X1^4*v^4*F^2*R^2-
4*M^4*X*v^4*X2*F^2*R^2+4*M^4*X1^2*F^6*R^2+8*M^3*X1^3*F^6*R^2+4*M^2*X1^4*F^6*R^2
+4*X2^2*X1^4*F^6*R^2-8*X2^2*R1^2*X1^2*F^2*R^2*v^2-8*X2^2*R1^2*M^2*F^2*R^2*v^2-
8*X2^2*R1^2*M^2*F^4*R^2-8*X2^2*R1^2*X1^2*F^4*R^2-
8*M^3*R1^2*X2*F^2*R^2*v^2+8*M^3*R1^2*X1*F^4*R^2+8*M^2*R1^2*X1^2*F^4*R^2-
16*M^3*R1^2*X1*F^3*R^2*v+4*M^2*R1^4*F^2*R^2+16*X2^2*R1^2*X1^2*F^3*R^2*v-
8*M^3*R1^2*X2*F^4*R^2-16*M^4*X1^2*F^5*v*R^2-16*M^4*X^3*F^5*v^3*X2-
16*X2^2*X1^4*F^5*v*R^2+24*X2^2*X1^4*F^4*v^2*R^2-16*X2^2*X1^4*F^3*v^3*R^2-
4*M^2*X*v^2*R1^2*R^2+4*M^4*X^3*v^4*X2*F^4-
8*M*X1*X*v^2*R1^2*R^2+8*X2^2*R1^2*M^2*X^2*F^4*v^2+8*X2^2*R1^2*M^2*X^2*F^6+8*M*R
1^4*X2*F^2*R^2+4*X2^2*R1^4*F^2*R^2+8*M^3*R1^2*X2*X^2*F^4*v^2+8*M^2*X*F*v*R1^2*R^
2+8*M^3*R1^2*X2*X^2*F^6+8*M^2*X1*F*v*R1^2*R^2-4*M*R1^4*R^2-4*X2*R1^4*R^2-
8*X2*X1^2*F*v*R1^2*R^2+4*X2^2*M^4*v^4*F^2*R^2-4*X2^2*M^4*F^8*X^2-
4*M^4*X*F^6*X2*R^2+4*X2*M^2*v^2*R1^2*R^2-
4*X2^2*M^4*v^4*X^2*F^4+4*X2*M^2*F^2*R1^2*R^2-4*M^2*X*F^2*R1^2*R^2-
8*X1^2*X*F^2*R1^2*R^2+4*X2*X1^2*F^2*R1^2*R^2-
4*M*X1^2*F^2*R1^2*R^2+8*M^4*X1*F^8*X2*X^2-
4*M^2*X1*F^2*R1^2*R^2+4*X2*X1^2*v^2*R1^2*R^2+24*X2*M*X1*X*F^2*v^2*R1^2*R^2+4*M^
2*X^2*F^3*v*R1^2*R^2-2*M^2*X^2*F^2*v^2*R1^2*R^2)^(1/2))]

2.B Symbolic Solution For v in Terms of F & C: -

V1=[1/2/(X^2*M^2*F^4*R2+2*X^2*M*X1*F^4*R2+X^2*X1^2*F^4*R2-2*X*M^2*F^2*R2*C-
4*X*M*X1*F^2*R2*C-X1^2*F^2*R2*R^2+X*X1*C^2*R+X*M^2*C^2*R+2*X*M*X1*C^2*R-
2*X*X1*F^2*R2*C+2*M*X1*R2*C^2+2*X*M^2*F^2*C*R1+4*X*M*X1*F^2*C*R+2*X*X1^2*F^2*
C*R-M^2*F^2*R2*R^2-2*M*X1*F^2*R2*R^2+M^2*R2*C^2+X1^2*R2*C^2)*(-
2*X^2*M*X1*F^5*R1-M^2*F^3*R^2*R1+X^2*M^2*F^5*R1+4*X*M*X1*F^3*C*R1-
2*M*X1*F*C^2*R1-
2*M*X1*F^3*R^2*R1+2*X*M^2*F^3*C*R1+M^2*F*C^2*R1+2*X*M^2*C^2*R*F+4*X*X1^2*F^3*
C*R+8*X*M*X1*F^3*C*R-2*M^2*F^3*R2*R^2-4*M*X1*F^3*R2*R^2+4*X*M*X1*C^2*R*F-
4*X*X1*F^3*R2*C+2*M^2*R2*C^2*F+4*M*X1*R2*C^2*F+2*X1^2*R2*C^2*F+2*X^2*M^2*F^5*R
2+2*X^2*X1^2*F^5*R2+4*X^2*M*X1*F^5*R2-4*X*M^2*F^3*R2*C+2*X*X1*C^2*R*F-
8*X*M*X1*F^3*R2*C-2*X1^2*F^3*R2*R^2-
(M^4*F^6*R^4*R1^2+X^4*M^4*F^10*R1^2+M^4*F^2*C^4*R1^2+4*M^2*R2^2*C^4*R1^2+4*X1^2*
R2^2*C^4*R1^2-32*X*M*X1*F^2*R2^2*C^3*R1^2+32*X*M*X1*F^4*R2^2*C*R^2*R1^2-
32*X^2*M*X1*F^2*R2*C^3*R*R1^2-
8*X1^2*F^2*R2^2*R^2*R1^2*C^2+24*X^2*M^2*F^4*R2^2*R1^2*C^2+4*X1^2*F^4*R2^2*R^4*R1^
2+8*X1^2*F^4*R2^2*R^2*X*C*R1^2-
4*X1^2*F^2*R2*R^3*X*C^2*R1^2+4*X*X1*C^4*R*R2*R1^2-
4*X*X1*C^2*R^3*F^2*R2*R1^2+4*X^3*X1*C^2*R*F^4*R2*R1^2-
16*X^2*X1*C^3*R*F^2*R2*R1^2+4*X^2*X1*C^4*R^2*R1^2-8*X^2*X1*C^3*R^2*F^2*R1^2-
16*X*M^2*F^2*R2^2*C^3*R1^2+16*X*M^2*F^4*R2^2*C*R^2*R1^2-
16*X^2*M^2*F^2*R2*C^3*R*R1^2+16*X^2*M^2*F^4*R2*C^2*R*R1^2-
8*X^2*M^2*F^6*R2^2*R^2*R1^2+4*X^4*M^2*F^8*R2^2*R1^2+48*X^2*M*X1*F^4*R2^2*R1^2*C^
2-16*X^2*M*X1*F^6*R2^2*R^2*R1^2+8*X^4*M*X1*F^8*R2^2*R1^2-
32*X^3*M*X1*F^6*R2^2*C*R1^2+16*X^3*M*X1*F^4*R2*C^2*R*R1^2+8*X^2*X1^2*F^4*R2^2*R
1^2*C^2-8*X^2*X1^2*F^6*R2^2*R^2*R1^2+4*X^4*X1^2*F^8*R2^2*R1^2-
8*X^3*X1^2*F^6*R2^2*C*R1^2+4*X^3*X1^2*F^4*R2*C^2*R*R1^2+4*M^2*X1^2*F^2*C^4*R1^2+
8*M^2*X1^2*F^4*C^2*R1^2*R^2-4*M^3*X1*F^2*C^4*R1^2+4*M^2*X1^2*F^6*R^4*R1^2-
4*X*M^4*F^4*C^3*R1^2+4*M^3*F^6*R^4*R1^2*X1+4*M^4*F^6*R^2*R1^2*X*C-
2*M^4*F^4*R^2*R1^2*C^2-4*X^3*M^4*F^8*R1^2*C+6*X^2*M^4*F^6*R1^2*C^2-
16*X*M^2*X1^2*F^4*C^3*R1^2-
16*X*M^2*X1^2*F^6*C*R1^2*R^2+16*X*M^3*X1*F^4*C^3*R1^2-

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16*X^3*M^2*X1^2*F^8*R1^2*C+24*X^2*M^2*X1^2*F^6*R1^2*C^2+8*X^2*M^2*X1^2*F^8*R1^2*
R^2+16*X^3*M^3*X1*F^8*R1^2*C-24*X^2*M^3*X1*F^6*R1^2*C^2-
2*M^4*F^8*R^2*R1^2*X^2+4*X^4*M^2*X1^2*F^10*R1^2-4*X^4*M^3*X1*F^10*R1^2-
8*M^2*F^2*R2^2*R^2*R1^2*C^2+4*M^2*F^4*R2^2*R^4*R1^2+8*M^2*F^4*R2*R^3*X*C*R1^2-
16*X^2*X1^2*F^4*C^2*R*R2*R1^2+8*X^2*X1^2*F^2*C^3*R^2*R1^2-
16*X^2*X1^2*F^4*C^2*R^2*R1^2-32*X^2*M*X1*F^4*C^2*R^2*R1^2-
8*X*M^2*F^4*C*R1^3*R2*R^2+8*X^3*M^2*F^6*C*R1^3*R2-
16*X^2*M^2*F^4*C^2*R1^3*R2+8*X^2*M^2*F^2*C^3*R1^3*R-16*X^2*M^2*F^4*C^2*R1^3*R-
16*M*X1*R2^2*C^2*F^2*R^2*R1^2+8*X*M^2*F^2*C^3*R1^3*R2+8*X*X1*F^4*R2^2*C*R^2*R1^
2-
8*X^3*X1*F^6*R2^2*C*R1^2+16*X^2*X1*F^4*R2^2*C^2*R1^2+16*X^2*X1*F^4*R2*C^2*R*R1^2
+8*M*X1*R2^2*C^4*R1^2+8*X*M^2*C^4*R*R2*R1^2-
8*X*M^2*C^2*R^3*F^2*R2*R1^2+4*X^2*M^2*C^4*R^2*R1^2-
8*X^2*M^2*C^3*R^2*F^2*R1^2+16*X*M*X1*C^4*R*R2*R1^2-
16*X*M*X1*C^2*R^3*F^2*R2*R1^2+8*X^2*M*X1*C^4*R^2*R1^2-8*X*X1*F^2*R2^2*C^3*R1^2-
8*X1^2*R2^2*C^3*X*F^2*R1^2+4*X1^2*R2*C^4*X*R*R1^2-
8*M^2*R2*C^3*X*F^2*R*R1^2+8*M*X1*F^4*R2^2*R^4*R1^2-
16*X^3*M^2*F^6*R2^2*C*R1^2+8*X^3*M^2*F^4*R2*C^2*R*R1^2-
8*X^3*M^2*F^6*R2*C*R*R1^2)^(1/2))]
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V2=[ 1/2/(-2*X2*M*X1*C^2+X2*X1^2*F^2*R^2+2*X*M*X1*C^2+X*M^2*C^2-M^2*C*R^2-
X1^2*C*R^2-X2*M^2*C^2-
X2*X1^2*C^2+M^2*X1*C^2+M*X1^2*C^2+2*X1^2*X*C^2+M*X1^2*X^2*F^4+4*X2*M*X1*X*F^
2*C-2*X2*M*X1*X^2*F^4-X2*X1^2*X^2*F^4+M^2*X1*X^2*F^4+2*X2*X1^2*X*F^2*C-
2*M*X1*C*R^2+2*X2*M*X1*F^2*R^2+2*X2*M^2*F^2*C-2*M*X1*X^2*F^2*C-X1^2*X^2*F^2*C-
2*M*X1^2*X*F^2*C+X2*M^2*F^2*R^2-M^2*X1*F^2*R^2-M*X1^2*F^2*R^2-2*M^2*X1*X*F^2*C-
M^2*X^2*F^2*C-X2*M^2*X^2*F^4)*(2*M*X1^2*X^2*F^5-2*X2*X1^2*X^2*F^5-
4*X2*M*X1*X^2*F^5+2*M^2*X1*C^2*F+2*M*X1^2*C^2*F+2*X*M^2*C^2*F+4*X*M*X1*C^2*F
+4*X1^2*X*C^2*F+4*X2*X1^2*X*F^3*C+8*X2*M*X1*X*F^3*C-2*M^2*C*R^2*F-
2*X1^2*C*R^2*F-2*X2*M^2*C^2*F-4*X2*M*X1*C^2*F-2*X2*X1^2*C^2*F+2*X2*M^2*F^3*R^2-
4*M*X1*C*R^2*F+2*X2*X1^2*F^3*R^2+4*X2*M*X1*F^3*R^2+4*X2*M^2*F^3*C-
2*M^2*X^2*F^3*C-2*X1^2*X^2*F^3*C-4*M*X1*X^2*F^3*C+2*M^2*X1*X^2*F^5-
4*M*X1^2*X*F^3*C-2*M^2*X1*F^3*R^2-2*M*X1^2*F^3*R^2-4*M^2*X1*X*F^3*C-
2*X2*M^2*X^2*F^5-2*(M^2*C*R^4*R1^2+X1^2*C*R^4*R1^2+2*X1^2*X^2*C^4*R1^2-
M^2*X1^2*C^4*R1^2+X2*X1^2*C^2*R1^2*R^2-M^2*X1*C^2*R1^2*R^2-
M^3*X1*C^4*R1^2+X2*M^2*C^2*R1^2*R^2-
X*M^2*C^2*R1^2*R^2+X2^2*X1^2*C^4*R1^2+X2^2*M^2*C^4*R1^2+X2*M^3*C^4*R1^2+M^3*C^
3*R^2*R1^2-X*M^3*C^4*R1^2-2*X*M*X1*C^2*R1^2*R^2+X2*X1^2*F^2*R^2*X*R1^2*C^2-
2*X2^2*X1^2*F^2*R^2*R1^2*C^2-2*X2^2*X1^2*F^6*R^2*X^2*R1^2-
X2*X1^2*F^4*R^2*X^2*R1^2*C+4*X2^2*X1^2*F^4*R^2*X*R1^2*C-
4*X2^2*M*X1*C^2*F^2*R1^2*R^2-4*X2*M*X1*X*F^2*C*R1^2*R^2+X^2*M^2*C^4*R1^2-
2*X1^2*X*C^2*R1^2*R^2-M*X1^2*X^2*F^4*R1^2*R^2-M*X1^2*C^2*R1^2*R^2-
M^2*X1*X^2*F^4*R1^2*R^2-2*X2*X1^2*X*F^2*C*R1^2*R^2-
2*X2*M^2*X1*X^2*F^6*R1^2*R^2+2*X2^2*M*X1*X^4*F^8*R1^2+4*X2*M*X1*X^4*F^6*R1^2*C
+X2*M^2*X1*X^4*F^8*R1^2+X2*X1^2*X^2*F^4*R1^2*R^2+8*X2^2*M*X1*X*F^4*C*R1^2*R^2+
4*X2*M^2*X1*X*F^4*C*R1^2*R^2-8*X2^2*M*X1*X^3*F^6*C*R1^2-
4*X2*M^2*X1*X^3*F^6*C*R1^2-3*X1^2*X^3*C^3*F^2*R1^2-
X1^2*X^3*C^2*M*F^4*R1^2+2*M^2*X1^2*X^2*F^6*R1^2*R^2-
M^2*X1^2*X^4*F^8*R1^2+4*M^2*X1^2*X^3*F^6*R1^2*C+2*M^3*X1*C^2*F^2*R1^2*R^2-
6*M^3*X1*C^2*X^2*F^4*R1^2+4*M^3*X1*C^3*X*F^2*R1^2-
M*X1^2*C^4*X*R1^2+2*M^2*X1^2*C^2*F^2*R1^2*R^2+2*M*X1^2*C^3*X^2*F^2*R1^2-
6*M^2*X1^2*C^2*X^2*F^4*R1^2+4*M^2*X1^2*C^3*X*F^2*R1^2-
3*X2*X1^2*C^4*X*R1^2+6*X2^2*X1^2*C^2*X^2*F^4*R1^2+8*X2*X1^2*C^3*X^2*F^2*R1^2-
4*X2^2*X1^2*C^3*X*F^2*R1^2-
2*X2*M^3*C^2*F^2*R1^2*R^2+2*X2^2*M^2*C^2*X^2*F^4*R1^2+2*X2*M^3*C^2*X^2*F^4*R1^2-
2*X2*M^3*C^3*X*F^2*R1^2-
2*X2^2*M^2*C^3*X*F^2*R1^2+X1^2*C^3*R^2*M*R1^2+X1^2*C^3*R^2*X2*R1^2-
X1^2*C*R^4*M*F^2*R1^2+X1^2*C^2*R^2*X^2*F^2*R1^2+X1^2*C*R^2*M*X^2*F^4*R1^2-
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X1^2*C^2*R^2*M*X*F^2*R1^2-2*X2^2*M^2*C^2*F^2*R1^2*R^2-M^3*C*R^4*F^2*R1^2-
M^2*C*R^2*X2*X^2*F^4*R1^2+M^2*C^2*R^2*X^2*F^2*R1^2-X1^2*C*R^4*X2*F^2*R1^2-
X1^2*C^3*R^2*X*R1^2-M^2*C^3*R^2*X*R1^2-2*X*M^2*C^4*X2*R1^2-
X*M^3*C^2*F^2*R1^2*R^2-4*X^3*M^2*C^2*X2*F^4*R1^2-2*X^3*M^2*C^3*F^2*R1^2-
3*X^3*M^3*C^2*F^4*R1^2+3*X^2*M^3*C^3*F^2*R1^2+4*X^2*M^2*C^3*X2*F^2*R1^2-
M^2*C*R^4*X2*F^2*R1^2-
3*X^3*M^2*X1*C^2*F^4*R1^2+3*X^2*M^2*X1*C^3*F^2*R1^2+2*X^2*M*X1*C^4*R1^2-
X*M^2*X1*C^4*R1^2-3*X*M^2*X1*C^2*F^2*R1^2*R^2-
4*X^3*M*X1*C^3*F^2*R1^2+2*M*X1*X^2*F^2*C*R1^2*R^2-2*X2*M^2*F^2*C*R1^2*R^2-
2*X2*M*X1*F^2*R^4*R1^2+2*M*X1*C*R^4*R1^2+2*M*X1*X^4*F^4*C^2*R1^2+2*X2*M^2*F^2*
C^3*X*R1^2-2*X2*M^3*F^2*C^3*R1^2-
2*X2^2*M^2*F^2*C^3*R1^2+2*X2*M^3*F^4*C*R1^2*R^2-2*X2^2*M^2*F^6*C*X^2*R1^2-
2*X2*M^2*F^4*C^2*X^2*R1^2-
2*X2*M^3*F^6*C*X^2*R1^2+4*X2*M^3*F^4*C^2*X*R1^2+4*X2^2*M^2*F^4*C^2*X*R1^2+2*X2^
2*M^2*F^4*C*R1^2*R^2+X2*M^2*X1*F^4*R^4*R1^2-
2*M*X1*C^3*R^2*X*R1^2+2*M^2*X1*C^3*R^2*R1^2+2*M*X1*C^3*R^2*X2*R1^2-
2*M^2*X1*C*R^4*F^2*R1^2-
2*M*X1*C*R^2*X2*X^2*F^4*R1^2+2*M*X1*C^2*R^2*X^2*F^2*R1^2+M^2*X1*C*R^2*X^2*F^4*
R1^2+2*X2^2*M*X1*F^4*R^4*R1^2-2*M*X1*C*R^4*X2*F^2*R1^2+M^2*X1*X^4*F^6*R1^2*C-
M^3*X1*X^4*F^8*R1^2+4*M^3*X1*X^3*F^6*R1^2*C+X2^2*X1^2*X^4*F^8*R1^2+X2*M^2*X^2*
F^4*R1^2*R^2+M^2*X^2*F^2*C*R1^2*R^2+2*M^2*X1*X*F^2*C*R1^2*R^2+M*X1^2*F^2*R^4*R1
^2+M^2*X1*F^2*R^4*R1^2-
X2*M^2*F^2*R^4*R1^2+2*M*X1^2*X*F^2*C*R1^2*R^2+X1^2*X^2*F^2*C*R1^2*R^2+X2*M^3*X
^4*F^8*R1^2-2*X2*M^3*X^3*F^6*R1^2*C-
2*X2^2*M^2*X^3*F^6*R1^2*C+2*M^2*X^4*F^6*C*X2*R1^2+M^2*X^4*F^4*C^2*R1^2+M^3*X^4
*F^6*C*R1^2+X2^2*M^2*X^4*F^8*R1^2-M^2*X1^2*F^4*R^4*R1^2-M^3*X1*F^4*R^4*R1^2-
4*M^3*X1*F^4*R^2*X*R1^2*C+X2*M^3*F^4*R^4*R1^2-2*X2^2*M^2*F^6*R^2*X^2*R1^2-
2*X2*M^3*F^6*R^2*X^2*R1^2+2*X2*M^3*F^4*R^2*X*R1^2*C+2*X2^2*M^2*F^4*R^2*X*R1^2*
C-
4*M^2*X1^2*X*F^4*C*R1^2*R^2+X2^2*M^2*F^4*R^4*R1^2+X1^2*X^4*F^4*C^2*R1^2+2*X2*M*
X1*C^2*R1^2*R^2-X2*X1^2*F^2*R^4*R1^2-
4*X2*M*X1*C^4*X*R1^2+X2*M^2*X1*C^4*R1^2+2*X2^2*M*X1*C^4*R1^2-
2*X2*M^2*X1*C^2*F^2*R1^2*R^2+12*X2^2*M*X1*C^2*X^2*F^4*R1^2+12*X2*M*X1*C^3*X^2*
F^2*R1^2+6*X2*M^2*X1*C^2*X^2*F^4*R1^2-4*X2*M^2*X1*C^3*X*F^2*R1^2-
8*X2^2*M*X1*C^3*X*F^2*R1^2+X2^2*X1^2*F^4*R^4*R1^2+2*X2*M*X1*X^2*F^4*R1^2*R^2-
12*X^3*M*X1*C^2*X2*F^4*R1^2+M^2*C^3*R^2*X2*R1^2-7*X1^2*X^3*C^2*X2*F^4*R1^2-
4*X2^2*M*X1*X^2*F^6*R1^2*R^2-
4*X2^2*X1^2*X^3*F^6*R1^2*C+2*M^3*X1*X^2*F^6*R1^2*R^2+2*X2*X1^2*X^4*F^6*R1^2*C)^(1
/2))]

3.B Symbolic Solution For M in Terms of v, F & C: -

M1=[1/2/(-F^4*R^2*R1-X^2*F^5*R1*v+X*R*C^2*F^2-2*X*R*C^2*F*v+X*R*C^2*v^2-
2*R2*C^2*F*v-
2*X^2*F^5*R2*v+X^2*F^4*R2*v^2+X^2*F^6*R2+F^3*R^2*R1*v+2*F^3*R2*R^2*v+R2*C^2*F^2-
F*C^2*R1*v-2*X*F^4*R2*C+F^2*C^2*R1-
2*X*F^2*R2*C*v^2+R2*C^2*v^2+X^2*F^6*R1+4*X*F^3*R2*C*v+2*X*F^2*C*R1*v^2-
F^2*R2*R^2*v^2-F^4*R2*R^2-2*X*F^3*C*R1*v)*(-2*X*X1*R*C^2*F^2+4*X*X1*R*C^2*F*v-
2*X*X1*R*C^2*v^2+4*X*X1*F^4*R2*C-8*X*X1*F^3*R2*C*v+4*X*X1*F^2*R2*C*v^2-
4*X*X1*F^4*C*R1+4*X*X1*F^3*C*R1*v-4*X*X1*F^4*R*C+8*X*X1*F^3*R*C*v-
4*X*X1*F^2*R*C*v^2+2*X1*F^2*C^2*R1-2*X1*F*C^2*R1*v-
2*X^2*X1*F^6*R2+4*X^2*X1*F^5*R2*v-2*X^2*X1*F^4*R2*v^2+2*X1*F^4*R^2*R1-
2*X1*F^3*R^2*R1*v+2*X1*F^4*R2*R^2-
4*X1*F^3*R2*R^2*v+2*X1*F^2*R2*R^2*v^2+2*X^2*X1*F^6*R1-2*X^2*X1*F^5*R1*v-
2*X1*R2*C^2*F^2+4*X1*R2*C^2*F*v-2*X1*R2*C^2*v^2-
2*(X1^2*F^4*C^4*R1^2+X1^2*F^8*R^4*R1^2+X^4*X1^2*F^12*R1^2+F^6*R^4*R1^3*R2+X^4*F^1
0*R2^2*R1^2+R2^2*C^4*F^2*R1^2+F^2*C^4*R1^3*R2+R2^2*C^4*v^2*R1^2+X^4*F^10*R1^3*R2+
F^6*R2^2*R^4*R1^2+X^2*X1^2*R^2*C^4*F^4-

4*X^2*X1^2*R^2*C^4*F^3*v+6*X^2*X1^2*R^2*C^4*F^2*v^2-
4*X^2*X1^2*R*C^3*F^6*R2+16*X^2*X1^2*R*C^3*F^5*R2*v-24*X^2*X1^2*R*C^3*F^4*R2*v^2-
2*X1^2*F^4*C^4*R1*X*R+6*X1^2*F^3*C^4*R1*X*R*v-
6*X1^2*F^2*C^4*R1*X*R*v^2+8*X1^2*F^6*C^3*R1*X*R2-
22*X1^2*F^5*C^3*R1*X*R2*v+18*X1^2*F^4*C^3*R1*X*R2*v^2-
8*X^2*X1^2*F^3*R^2*C^3*v^3+16*X^2*X1^2*F^5*R*C^2*v^3*R2+2*X^2*X1^2*F^2*R^2*C^3*v^
4-4*X^2*X1^2*F^4*R*C^2*v^4*R2-4*X^2*X1^2*F^8*R*C^2*R2+16*X^2*X1^2*F^7*R*C^2*R2*v-
24*X^2*X1^2*F^6*R*C^2*R2*v^2-
4*X^2*X1^2*F^3*C^3*R1*v^3*R+14*X^2*X1^2*F^5*C^2*R1*v^3*R2+2*X^2*X1^2*F^6*R^2*C^3-
8*X^2*X1^2*F^5*R^2*C^3*v+4*X^2*X1^2*F^6*C^3*R1*R-
12*X^2*X1^2*F^5*C^3*R1*R*v+12*X^2*X1^2*F^4*C^3*R1*R*v^2-
14*X^2*X1^2*F^8*C^2*R1*R2+42*X^2*X1^2*F^7*C^2*R1*R2*v-
42*X^2*X1^2*F^6*C^2*R1*R2*v^2+4*X^2*X1^2*F^4*R2^2*C^2*v^4+12*X^2*X1^2*F^4*R^2*C^3
*v^2+4*X^2*X1^2*F^8*R2^2*C^2-
16*X^2*X1^2*F^7*R2^2*C^2*v+24*X^2*X1^2*F^6*R2^2*C^2*v^2-
16*X^2*X1^2*F^5*R2^2*C^2*v^3+16*X^2*X1^2*R*C^3*F^3*v^3*R2+X^2*X1^2*R^2*C^4*v^4-
4*X^2*X1^2*R*C^3*v^4*F^2*R2-4*X^2*X1^2*R^2*C^4*F*v^3-
X1^2*F^2*R2*R^3*v^4*X*C^2+2*X1^2*F^4*R2^2*R^2*v^4*X*C+4*X1^2*F^3*R2*R^3*v^3*X*C^2
-8*X1^2*F^5*R2^2*R^2*v^3*X*C-6*X1^2*F^5*R^2*R1*v^3*X*R2*C-
X1^2*F^6*R2*R^3*X*C^2+4*X1^2*F^5*R2*R^3*X*C^2*v-
6*X1^2*F^4*R2*R^3*X*C^2*v^2+2*X1^2*F^8*R2^2*R^2*X*C-
8*X1^2*F^7*R2^2*R^2*X*C*v+12*X1^2*F^6*R2^2*R^2*X*C*v^2-
2*X1^2*F^7*R^2*R1*X*R2*C*v+6*X1^2*F^6*R^2*R1*X*R2*C*v^2+2*X1^2*F^3*R^3*R1*v^3*X*
C^2-2*X1^2*F^6*R^3*R1*X*C^2+6*X1^2*F^5*R^3*R1*X*C^2*v-
6*X1^2*F^4*R^3*R1*X*C^2*v^2+8*X^3*X1^2*F^7*R2^2*v^3*C+X^3*X1^2*F^4*R2*v^4*R*C^2-
2*X^3*X1^2*F^6*R2^2*v^4*C-4*X^3*X1^2*F^5*R2*v^3*R*C^2-
2*X1^2*F^3*C^3*R1*v^3*X*R2+X^3*X1^2*F^8*R2*R*C^2-
4*X^3*X1^2*F^7*R2*R*C^2*v+6*X^3*X1^2*F^6*R2*R*C^2*v^2-
2*X^3*X1^2*F^10*R2^2*C+8*X^3*X1^2*F^9*R2^2*C*v-
12*X^3*X1^2*F^8*R2^2*C*v^2+2*X1^2*F*C^4*R1*v^3*X*R+X1^2*R2*C^4*v^4*X*R-
2*X1^2*R2^2*C^3*v^4*X*F^2-4*X1^2*R2*C^4*F*v^3*X*R+8*X1^2*R2^2*C^3*F^3*v^3*X-
2*X1^2*R2^2*C^3*F^6*X+8*X1^2*R2^2*C^3*F^5*X*v-12*X1^2*R2^2*C^3*F^4*X*v^2-
2*X^3*X1^2*F^8*R1*R*C^2+6*X^3*X1^2*F^7*R1*R*C^2*v-
6*X^3*X1^2*F^6*R1*R*C^2*v^2+8*X^3*X1^2*F^10*R1*R2*C-
22*X^3*X1^2*F^9*R1*R2*C*v+18*X^3*X1^2*F^8*R1*R2*C*v^2+2*X^3*X1^2*F^5*R1*v^3*R*C^
2-2*X^3*X1^2*F^7*R1*v^3*R2*C+X1^2*R2*C^4*F^4*X*R-
4*X1^2*R2*C^4*F^3*X*R*v+6*X1^2*R2*C^4*F^2*X*R*v^2+6*X^2*X1^2*F^8*C^2*R1^2-
12*X^2*X1^2*F^7*C^2*R1^2*v+8*X^2*X1^2*F^8*C^2*R1*R-
20*X^2*X1^2*F^7*C^2*R1*R*v+12*X^2*X1^2*F^6*C^2*R1*R*v^2-
4*X*X1^2*F^6*C^3*R1^2+8*X*X1^2*F^5*C^3*R1^2*v+6*X^2*X1^2*F^6*C^2*R1^2*v^2+4*X^2*X
1^2*F^5*C^2*R1*v^3*R-4*X*X1^2*F^4*C^3*R1^2*v^2+4*X^2*X1^2*F^8*R^2*C^2-
16*X^2*X1^2*F^7*R^2*C^2*v+24*X^2*X1^2*F^6*R^2*C^2*v^2-
6*X*X1^2*F^6*R*C^3*R1+18*X*X1^2*F^5*R*C^3*R1*v-16*X^2*X1^2*F^5*R^2*C^2*v^3-
18*X*X1^2*F^4*R*C^3*v^2*R1+4*X^2*X1^2*F^4*R^2*C^2*v^4+6*X*X1^2*F^3*R*C^3*v^3*R1-
2*X1^2*F^3*C^4*R1^2*v+X1^2*F^2*C^4*R1^2*v^2+2*X^2*X1^2*F^10*R2*R^2*R1-
6*X^2*X1^2*F^9*R2*R^2*R1*v-3*X^4*X1^2*F^12*R2*R1+9*X^4*X1^2*F^11*R2*R1*v-
4*X*X1^2*F^8*C*R1^2*R^2+8*X*X1^2*F^7*C*R1^2*R^2*v-
4*X^3*X1^2*F^10*C*R1^2+8*X^3*X1^2*F^9*C*R1^2*v-4*X*X1^2*F^6*C*R1^2*v^2*R^2-
4*X^3*X1^2*F^8*C*R1^2*v^2+2*X^3*X1^2*F^10*R*C*R2-
8*X^3*X1^2*F^9*R*C*R2*v+12*X^3*X1^2*F^8*R*C*R2*v^2-
2*X*X1^2*F^8*R^3*C*R1+6*X*X1^2*F^7*R^3*C*R1*v-
2*X*X1^2*F^8*R^3*C*R2+8*X*X1^2*F^7*R^3*C*R2*v-12*X*X1^2*F^6*R^3*C*R2*v^2-
6*X^3*X1^2*F^10*R*C*R1+18*X^3*X1^2*F^9*R*C*R1*v+2*X*X1^2*F^6*R*C^3*R2-
8*X*X1^2*F^5*R*C^3*R2*v+12*X*X1^2*F^4*R*C^3*R2*v^2-8*X^3*X1^2*F^7*R*C*v^3*R2-
6*X*X1^2*F^6*R^3*C*v^2*R1+8*X*X1^2*F^5*R^3*C*v^3*R2-18*X^3*X1^2*F^8*R*C*v^2*R1-
8*X*X1^2*F^3*R*C^3*v^3*R2+2*X^3*X1^2*F^6*R*C*v^4*R2+2*X*X1^2*F^5*R^3*C*v^3*R1-
2*X*X1^2*F^4*R^3*C*v^4*R2+6*X^3*X1^2*F^7*R*C*v^3*R1+2*X*X1^2*F^2*R*C^3*v^4*R2+2*
X1^2*F^6*C^2*R1^2*R^2-4*X1^2*F^5*C^2*R1^2*R^2*v+2*X1^2*F^6*C^2*R1*R2*R^2-
6*X1^2*F^5*C^2*R1*R2*R^2*v+6*X1^2*F^4*C^2*R1*R2*R^2*v^2-
3*X1^2*F^4*C^4*R1*R2+9*X1^2*F^3*C^4*R1*R2*v-

9*X1^2*F^2*C^4*R1*R2*v^2+2*X1^2*F^4*C^2*R1^2*v^2*R^2-
2*X1^2*F^3*C^2*R1*v^3*R2*R^2+3*X1^2*F*C^4*R1*v^3*R2-
2*F^4*R^2*R1^3*R2*C^2+F^6*R^3*R1*X*X1*C^2-
3*F^5*R^3*R1*X*X1*C^2*v+3*F^4*R^3*R1*X*X1*C^2*v^2+6*F^7*R^2*R1*X*X1*R2*C*v+6*X^
2*X1^2*F^8*R2*v^2*R^2*R1-9*X^4*X1^2*F^10*R2*v^2*R1-
2*X^2*X1^2*F^7*R2*v^3*R^2*R1+3*X^4*X1^2*F^9*R2*v^3*R1-
2*X1^2*F^7*R^4*R1^2*v+X1^2*F^8*R^4*R1*R2-
3*X1^2*F^7*R^4*R1*R2*v+3*X1^2*F^6*R^4*R1*R2*v^2+2*X1^2*F^10*R^2*R1^2*X^2-
4*X1^2*F^9*R^2*R1^2*X^2*v+X1^2*F^6*R^4*R1^2*v^2-
X1^2*F^5*R^4*R1*v^3*R2+2*X1^2*F^8*R^2*R1^2*v^2*X^2-
2*X^4*X1^2*F^11*R1^2*v+X^4*X1^2*F^10*R1^2*v^2-6*F^6*R^2*R1*X*X1*R2*C*v^2-
2*F^8*R^2*R1^3*X^2*R2+2*F^6*R^2*R1^3*X*R2*C+2*F^6*R^3*R1^3*X*C-
F^4*R^3*R1^3*X*C^2+2*X^2*F^5*R1^3*v*R2*C^2+3*X^3*F^7*R1*v*X1*R*C^2-
3*X^3*F^6*R1*v^2*X1*R*C^2-2*F^8*R^2*R1*X*X1*R2*C+X^3*F^5*R1*v^3*X1*R*C^2-
6*X^3*F^9*R1*v*X1*R2*C+6*X^3*F^8*R1*v^2*X1*R2*C-2*X^3*F^7*R1*v^3*X1*R2*C-
X^4*F^9*R1^3*v*R2+2*X^3*F^7*R1^3*v*R*C-
X^3*F^5*R1^3*v*R*C^2+2*X^2*F^7*R1^3*v*R2*R^2+2*X*R*C^4*F^2*R2*R1^2-
X^2*R^2*C^4*F^4*X1+4*X^2*R^2*C^4*F^3*X1*v-
6*X^2*R^2*C^4*F^2*X1*v^2+4*X^2*R*C^3*F^6*X1*R2-
16*X^2*R*C^3*F^5*X1*R2*v+24*X^2*R*C^3*F^4*X1*R2*v^2+2*X^3*R*C^2*F^6*R2*R1^2-
4*X^2*R*C^3*F^4*R2*R1^2-2*X^2*R^2*C^3*F^4*R1^2+X^2*R^2*C^4*F^2*R1^2-
2*X*R^3*C^2*F^4*R2*R1^2-4*X*R*C^4*F*v*R2*R1^2+4*X^2*R^2*C^4*F*v^3*X1-
16*X^2*R*C^3*F^3*v^3*X1*R2-
4*X^3*R*C^2*F^5*v*R2*R1^2+8*X^2*R*C^3*F^3*v*R2*R1^2+4*X^2*R^2*C^3*F^3*v*R1^2-
2*X^2*R^2*C^4*F*v*R1^2+4*X*R^3*C^2*F^3*v*R2*R1^2+2*X*R*C^4*v^2*R2*R1^2-
X^2*R^2*C^4*v^4*X1+4*X^2*R*C^3*v^4*X1*F^2*R2+2*X^3*R*C^2*v^2*F^4*R2*R1^2-
4*X^2*R*C^3*v^2*F^2*R2*R1^2-2*X^2*R^2*C^3*v^2*F^2*R1^2+X^2*R^2*C^4*v^2*R1^2-
2*X*R^3*C^2*v^2*F^2*R2*R1^2-2*R2^2*C^4*F*v*R1^2+4*R2*C^4*F^3*v*X*X1*R-
6*R2*C^4*F^2*v^2*X*X1*R+4*R2*C^4*F*v^3*X*X1*R-
8*R2^2*C^3*F^5*v*X*X1+12*R2^2*C^3*F^4*v^2*X*X1-8*R2^2*C^3*F^3*v^3*X*X1-
12*R2^2*C^2*F^5*v*X^2*R1^2+8*R2^2*C^3*F^3*v*X*R1^2+4*R2*C^3*F^3*v*X*R*R1^2+4*R2^2
*C^2*F^3*v*R^2*R1^2+4*X^3*F^7*R2*v*X1*R*C^2-
6*X^3*F^6*R2*v^2*X1*R*C^2+4*X^3*F^5*R2*v^3*X1*R*C^2-
8*X^3*F^9*R2^2*v*X1*C+12*X^3*F^8*R2^2*v^2*X1*C-8*X^3*F^7*R2^2*v^3*X1*C-
2*X^4*F^9*R2^2*v*R1^2+8*X^3*F^7*R2^2*v*C*R1^2+4*X^3*F^7*R2*v*R*C*R1^2+4*X^2*F^7*R
2^2*v*R^2*R1^2+6*X^2*F^4*R2^2*v^2*R1^2*C^2-
X^3*F^4*R2*v^4*X1*R*C^2+2*X^3*F^6*R2^2*v^4*X1*C+X^4*F^8*R2^2*v^2*R1^2-
4*X^3*F^6*R2^2*v^2*C*R1^2-2*X^3*F^6*R2*v^2*R*C*R1^2-
2*X^2*F^6*R2^2*v^2*R^2*R1^2+6*X^2*F^6*R2^2*R1^2*C^2-
X^3*F^8*R2*X1*R*C^2+2*X^3*F^10*R2^2*X1*C-4*X^3*F^8*R2^2*C*R1^2-
2*X^3*F^8*R2*R*C*R1^2-2*X^2*F^8*R2^2*R^2*R1^2+2*F^3*R^2*R1^3*v*R2*C^2-
F^3*R^3*R1*v^3*X*X1*C^2+2*F^5*R^2*R1*v^3*X*X1*R2*C-
2*F^5*R^3*R1^3*v*X*C+F^3*R^3*R1^3*v*X*C^2-F^5*R^4*R1^3*v*R2-
4*F^5*R2*R^3*v*X*X1*C^2+6*F^4*R2*R^3*v^2*X*X1*C^2-
4*F^3*R2*R^3*v^3*X*X1*C^2+8*F^7*R2^2*R^2*v*X*X1*C-
12*F^6*R2^2*R^2*v^2*X*X1*C+8*F^5*R2^2*R^2*v^3*X*X1*C-8*F^5*R2^2*R^2*v*X*C*R1^2-
4*F^5*R2*R^3*v*X*C*R1^2-2*F^5*R2^2*R^4*v*R1^2-
R2*C^4*F^4*X*X1*R+2*R2^2*C^3*F^6*X*X1-4*R2^2*C^3*F^4*X*R1^2-
2*R2*C^3*F^4*X*R*R1^2-2*R2^2*C^2*F^4*R^2*R1^2-
F*C^4*R1^3*v*R2+3*F^3*C^4*R1*v*X*X1*R-
3*F^2*C^4*R1*v^2*X*X1*R+F*C^4*R1*v^3*X*X1*R-
6*F^5*C^3*R1*v*X*X1*R2+6*F^4*C^3*R1*v^2*X*X1*R2-
2*F^3*C^3*R1*v^3*X*X1*R2+2*F^3*C^3*R1^3*v*X*R-F*C^4*R1^3*v*X*R-
4*X^2*F^8*R2^2*C^2*X1+16*X^2*F^7*R2^2*C^2*X1*v-
24*X^2*F^6*R2^2*C^2*X1*v^2+4*X^2*F^6*R2*C^2*R*R1^2+4*X*F^6*R2^2*C*R^2*R1^2-
F^4*C^4*R1*X*X1*R+2*F^6*C^3*R1*X*X1*R2+2*F^6*C^2*R1^3*X^2*R2-
2*F^4*C^3*R1^3*X*R2-2*F^4*C^3*R1^3*X*R+F^2*C^4*R1^3*X*R-
4*X*F^2*R2^2*C^3*v^2*R1^2+16*X^2*F^5*R2^2*C^2*v^3*X1-
4*X^2*F^4*R2^2*C^2*v^4*X1+4*X^2*F^4*R2*C^2*v^2*R*R1^2+4*X*F^4*R2^2*C*v^2*R^2*R1^2
-R2*C^4*v^4*X*X1*R+2*R2^2*C^3*v^4*X*X1*F^2-2*R2*C^3*v^2*X*F^2*R*R1^2-

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2*R2^2*C^2*v^2*F^2*R^2*R1^2-X^3*F^8*R1*X1*R*C^2+2*X^3*F^10*R1*X1*R2*C-
2*X^3*F^8*R1^3*R2*C-2*X^3*F^8*R1^3*R*C+X^3*F^6*R1^3*R*C^2-
8*X^2*F^5*R2*C^2*v*R*R1^2+2*X*F^2*C^3*R1^3*v^2*R2-2*X*F^2*C^3*R1*v^4*X1^2*R2-
4*X^2*F^4*C^2*R1*v^4*X1^2*R-6*X^2*F^4*C^3*R1*v^2*X1*R+6*X^2*F^3*C^3*R1*v^3*X1*R-
2*X^2*F^2*C^3*R1*v^4*X1*R+2*X*F^4*C*R1*v^4*X1^2*R2*R^2+12*X^2*F^6*C^2*R1*v^2*X1*
R2-12*X^2*F^5*C^2*R1*v^3*X1*R2+4*X^2*F^4*C^2*R1*v^4*X1*R2-
2*X^3*F^6*C*R1*v^4*X1^2*R2+2*X^3*F^6*C*R1^3*v^2*R2-4*X^2*F^4*C^2*R1^3*v^2*R2-
4*X^2*F^4*C^2*R1^3*v^2*R+2*X^2*F^2*C^3*R1^3*v^2*R-
2*X*F^4*C*R1^3*v^2*R2*R^2+F^2*R2*R^3*v^4*X*X1*C^2-
2*F^4*R2^2*R^2*v^4*X*X1*C+2*F^4*R2*R^3*v^2*X*C*R1^2+F^4*R2^2*R^4*v^2*R1^2+F^6*R2*
R^3*X*X1*C^2-
2*F^8*R2^2*R^2*X*X1*C+2*F^6*R2*R^3*X*C*R1^2+2*X^2*F^5*C^3*R1*v*X1*R-
4*X^2*F^7*C^2*R1*v*X1*R2+4*X^2*F^5*C^2*R1^3*v*R-2*X^2*F^3*C^3*R1^3*v*R)^(1/2))]

M2=[ 1/2/(2*X*C^2*F*v+2*X1*C^2*F*v-X*C^2*v^2-X1*C^2*F^2-X1*C^2*v^2+X2*C^2*v^2-
2*R^2*C*F*v-2*X2*C^2*F*v+R^2*C*F^2+R^2*C*v^2-X2*F^4*R^2+2*X1*X*F^4*C-
2*X^2*F^3*C*v+X^2*F^2*C*v^2-4*X1*X*F^3*C*v+2*X1*X*F^2*C*v^2+X^2*F^4*C+X1*F^4*R^2-
X*C^2*F^2+2*X2*F^3*R^2*v-X2*F^2*R^2*v^2-2*X2*F^4*C-2*X2*F^2*C*v^2-
2*X1*F^3*R^2*v+X1*F^2*R^2*v^2+4*X2*F^3*C*v+X2*C^2*F^2+2*X1*X^2*F^5*v-
X1*X^2*F^4*v^2-X1*X^2*F^6+X2*X^2*F^6-2*X2*X^2*F^5*v+X2*X^2*F^4*v^2)*(-(-
8*X2^2*F^5*R^4*v*R1^2+4*X^4*F^6*C^2*R1^2-
4*X^2*F^6*C*X2*R1^2*R^2+8*X^4*F^8*C*X2*R1^2-
8*X2*F^6*C^2*X^2*R1^2+4*X2^2*F^4*R^4*v^2*R1^2-
32*X2^2*F^3*C^3*v^3*X1^2+8*X2^2*F^2*C^3*v^4*X1^2-
32*X2^2*F^5*C^3*X1^2*v+48*X2^2*F^4*C^3*X1^2*v^2+4*X1*F^6*R^2*X^2*R1^2*C+X^4*F^8*R
1^4+X1^4*C^4*F^4+8*X2*F^6*C^2*X1^2*R^2-
16*X2*F^6*C^3*X1^2*X+8*X2*F^8*C^2*X1^2*X^2+F^4*R1^4*R^4+8*X2^2*F^6*C*R1^2*R^2-
8*X2^2*F^8*C*X^2*R1^2-8*X2^2*F^6*R^2*v^2*X^2*R1^2-
4*X2*F^4*R^2*v^2*X^2*R1^2*C+16*X2^2*F^7*R^2*v*X^2*R1^2+4*X2^2*X^4*F^8*v^2*R1^2-
8*X2^2*X^4*F^9*v*R1^2+8*X2*F^5*R^2*v*X^2*R1^2*C+64*X2*F^5*C^3*X1^2*X*v+8*X2^2*F^6
*C^3*X1^2+X1^4*F^8*R^4-
8*X2^2*F^4*C*v^4*X1^2*R^2+8*X2*F^2*C^2*v^4*X1^2*R^2+X1^4*C^4*v^4-
32*X2*F^3*C^2*v^3*X1^2*R^2-
16*X2*F^2*C^3*v^4*X1^2*X+64*X2*F^3*C^3*v^3*X1^2*X+8*X2^2*F^4*C*v^2*R1^2*R^2-
8*X2^2*F^6*C*v^2*X^2*R1^2-8*X2*F^4*C^2*v^2*X^2*R1^2+48*X2*F^4*C^2*X1^2*R^2*v^2-
32*X2*F^5*C^2*X1^2*R^2*v-96*X2*F^4*C^3*X1^2*X*v^2+X^4*X1^4*F^12-
16*X2^2*F^5*C*v*R1^2*R^2+16*X2^2*F^7*C*v*X^2*R1^2+16*X2*F^5*C^2*v*X^2*R1^2+6*X^4*
X1^4*F^10*v^2-4*X^4*X1^4*F^11*v-4*X^4*X1^4*F^9*v^3-
4*X^3*X1^3*F^8*C^2+X^4*X1^4*F^8*v^4-4*X^2*X1^2*C^4*v^4-
4*X^2*X1^2*C^4*F^4+4*X^3*X1^2*C^3*F^6+2*X^4*X1^2*F^10*R1^2-
2*X^2*X1^4*F^10*R^2+6*X^2*X1^4*F^8*C^2-4*X^3*X1^4*F^10*C-4*X*X1^3*C^4*v^4-
4*X*X1^3*C^4*F^4+8*X^2*X1^3*C^3*F^6-4*X*F^2*R1^4*C^3-2*X^2*F^6*R1^4*R^2-
2*F^2*R1^4*R^2*C^2+2*F^6*R1^2*R^4*X1^2+2*R1^2*C^4*X1^2*F^2+2*R1^2*C^4*X1^2*v^2+6*
X1^4*F^6*R^4*v^2-4*X1^4*F^5*R^4*v^3-4*X1^4*F^7*R^4*v+X1^4*F^4*R^4*v^4-
2*X1^4*F^6*R^2*C^2+6*X1^4*C^4*F^2*v^2-4*X1^4*C^4*F^3*v-
4*X1^4*C^4*F*v^3+16*X^3*X1^4*F^7*v^3*C-24*X^3*X1^4*F^8*v^2*C+16*X^3*X1^4*F^9*v*C-
8*X^3*X1^2*F^8*R1^2*C-4*X^2*X1^2*F^8*R1^2*R^2+12*X^2*X1^2*F^6*R1^2*C^2-
8*X^3*X1^2*F^6*v^2*R1^2*C+2*X^4*X1^2*F^8*v^2*R1^2-
4*X^2*X1^2*F^6*v^2*R1^2*R^2+12*X^2*X1^2*F^4*v^2*R1^2*C^2-
2*X^2*X1^4*F^6*v^4*R^2+6*X^2*X1^4*F^4*v^4*C^2-
4*X^3*X1^4*F^6*v^4*C+32*X2^2*X1^2*C^3*F^3*v^3*X-
8*X1*X^4*F^6*C*v^2*R1^2+32*X2*X1*X^3*F^8*C*R1^2-
32*X^2*C^3*F^3*v*X2*R1^2+8*X1*C^2*F*v*R1^2*R^2-4*X*C^2*v^2*R1^2*R^2-
8*X^4*F^8*R1^2*X2*X1*v^2-
4*X1^4*C^3*F^6*X+4*X^2*C^4*v^2*R1^2+4*X2^2*C^4*v^2*R1^2+4*R^4*C*F^2*R1^2+4*R^4*C*
v^2*R1^2-4*X2*F^4*R^4*R1^2+4*X2^2*F^6*R^4*R1^2-
8*X^3*F^4*C^3*R1^2+4*X1*F^4*R^4*R1^2+4*X^2*C^4*F^2*R1^2-
8*X2^2*F^4*C^3*R1^2+4*X2^2*C^4*F^2*R1^2-4*X1^4*C^3*v^4*X*F^2+8*X*C^2*F*v*R1^2*R^2-
8*X^2*C^4*F*v*R1^2+16*X*C^4*F*v*X2*R1^2-
8*X*C^4*v^2*X2*R1^2+16*X^2*C^3*v^2*X2*F^2*R1^2-4*X1*C^2*F^2*R1^2*R^2-
```

4*X1*C^2*v^2*R1^2*R^2+4*X2*C^2*v^2*R1^2*R^2-8*X2^2*C^3*v^2*X*F^2*R1^2-
8*R^4*C*F*v*R1^2+8*R^2*C^3*F*v*X*R1^2-8*R^2*C^3*F*v*X2*R1^2-8*X2*C^2*F*v*R1^2*R^2-
8*X2^2*C^4*F*v*R1^2+16*X2^2*C^3*F^3*v*X*R1^2-
4*R^2*C^3*F^2*X*R1^2+4*R^2*C^3*F^2*X2*R1^2-
4*R^2*C^3*v^2*X*R1^2+4*R^2*C^3*v^2*X2*R1^2-
8*X2^2*F^4*R^2*R1^2*C^2+8*X2^2*F^6*R^2*X*R1^2*C+8*X1*X*F^4*C*R1^2*R^2-
8*X^2*F^3*C*v*R1^2*R^2+16*X^3*F^3*C^3*v*R1^2+32*X^3*F^5*C^2*v*X2*R1^2+4*X^2*F^2*C
*v^2*R1^2*R^2-8*X^3*F^2*C^3*v^2*R1^2-16*X^3*F^4*C^2*v^2*X2*R1^2-
16*X1*X*F^3*C*v*R1^2*R^2+8*X1*X*F^2*C*v^2*R1^2*R^2+4*X^2*F^4*C*R1^2*R^2+16*X^2*F
^4*C^3*X2*R1^2-16*X^3*F^6*C^2*X2*R1^2+8*X2^2*C^2*v^2*X^2*F^4*R1^2-
8*X2^2*C^2*v^2*F^2*R1^2*R^2+4*X2^2*X^4*F^10*R1^2-
4*X^3*X1^3*F^4*v^4*C^2+R1^4*C^4+16*X^3*X1^3*F^5*v^3*C^2+16*X^3*X1^3*F^7*v*C^2-
24*X^3*X1^3*F^6*v^2*C^2-8*X2^2*X1^2*C^3*F^6*X+32*X2*X1*C^3*F^4*X*R1^2-
48*X2*X1*C^2*F^6*X^2*R1^2+16*X2*X1*C^2*F^4*R1^2*R^2-
8*X2*X1*C^4*F^2*R1^2+16*X2^2*X1^2*C^2*F^8*X^2-64*X2^2*X1^2*C^2*F^7*X^2*v-
64*X2*X1*C^3*F^3*v*X*R1^2+96*X2*X1*C^2*F^5*v*X^2*R1^2-
32*X2*X1*C^2*F^3*v*R1^2*R^2+16*X2*X1*C^4*F*v*R1^2+4*X1*R^4*C*F^4*R1^2-
4*X1*R^2*C^3*F^2*R1^2-
24*X^2*X1*C^3*F^4*R1^2+24*X^3*X1*C^2*F^6*R1^2+8*X*X1*C^4*F^2*R1^2-
16*X*X1^3*C^2*F^5*R^2*v+4*X*X1^3*C^2*F^6*R^2+16*X*X1^3*C^4*F^3*v-
32*X^2*X1^3*C^3*F^5*v-4*X^3*X1^2*C^2*F^8*X2+48*X^2*X1*C^3*F^3*v*R1^2-
48*X^3*X1*C^2*F^5*v*R1^2-16*X*X1*C^4*F*v*R1^2-
16*X^3*X1^2*C^3*v^3*F^3+4*X^3*X1^2*C^3*v^4*F^2-
24*X^2*X1*C^3*v^2*F^2*R1^2+24*X^3*X1*C^2*v^2*F^4*R1^2+8*X*X1*C^4*v^2*R1^2-
16*X*X1^3*C^2*v^3*F^3*R^2+4*X*X1^3*C^2*v^4*F^2*R^2+24*X*X1^3*C^2*v^2*F^4*R^2+16*X
*X1^3*C^4*v^3*F-24*X*X1^3*C^4*v^2*F^2+8*X^2*X1^3*C^3*v^4*F^2-
32*X^2*X1^3*C^3*v^3*F^3+48*X^2*X1^3*C^3*v^2*F^4-
4*X^3*X1^2*C^2*v^4*X2*F^4+16*X^3*X1^2*C^2*v^3*X2*F^5+16*X^3*X1^2*F^7*v*R1^2*C-
4*X^4*X1^2*F^9*v*R1^2+8*X^2*X1^2*F^7*v*R1^2*R^2-24*X^2*X1^2*F^5*v*R1^2*C^2-
12*X^2*X1^4*F^8*v^2*R^2+8*X^2*X1^4*F^7*v^3*R^2+8*X^2*X1^4*F^9*v*R^2+36*X^2*X1^4*F^
6*v^2*C^2-24*X^2*X1^4*F^7*v*C^2-
24*X^2*X1^4*F^5*v^3*C^2+96*X2^2*X1^2*X^2*F^6*C^2*v^2-
64*X2^2*X1^2*X^2*F^5*C^2*v^3+16*X2^2*X1^2*X^2*F^4*C^2*v^4+8*X2^2*X1^2*F^4*R^2*v^4*
X*C+16*X1^2*X^3*F^7*C^2*X2*v-24*X1^2*X^3*F^6*C^2*X2*v^2-
32*X2^2*X1^2*F^5*R^2*v^3*X*C-8*X2^2*X1^2*C^3*v^4*X*F^2-
32*X2^2*X1^2*F^7*R^2*X*C*v+48*X2^2*X1^2*F^6*R^2*X*C*v^2+8*X2*X1^2*C^3*F^6*X^2+32
*X2^2*X1^2*C^3*F^5*X*v-48*X2^2*X1^2*C^3*F^4*X*v^2-
32*X^2*X1^2*C^3*F^5*X2*v+48*X^2*X1^2*C^3*F^4*X2*v^2-16*X^3*X1^2*C^3*F^5*v-
24*X^2*X1^2*C^4*v^2*F^2+16*X^2*X1^2*C^4*v^3*F+24*X*X1^2*C^4*v^2*X2*F^2-
16*X*X1^2*C^4*v^3*X2*F+24*X*X1^2*C^3*v^2*R^2*F^2-
16*X*X1^2*C^3*v^3*R^2*F+4*X*X1^2*C^3*v^4*R^2+4*X*X1^2*C^4*v^4*X2-
72*X*X1^2*C^2*v^2*X2*F^4*R^2+48*X*X1^2*C^2*v^3*X2*F^3*R^2-
12*X*X1^2*C^2*v^4*X2*F^2*R^2+24*X^3*X1^2*C^3*v^2*F^4-
32*X^2*X1^2*C^3*v^3*X2*F^3+8*X^2*X1^2*C^3*v^4*X2*F^2+16*X^2*X1^2*C^4*F^3*v+4*X*X1
^2*C^4*F^4*X2-16*X*X1^2*C^4*F^3*X2*v+4*X*X1^2*C^3*F^4*R^2-16*X*X1^2*C^3*F^3*R^2*v-
12*X*X1^2*C^2*F^6*X2*R^2+48*X*X1^2*C^2*F^5*X2*R^2*v+4*X1^4*F^8*R^2*X*C+16*X1^4*C
^3*F^3*v^3*X-24*X1^4*C^3*F^4*v^2*X+16*X1^4*C^3*F^5*v*X-
12*X1^4*F^4*R^2*v^2*C^2+8*X1^4*F^5*R^2*v*C^2+8*X1^4*F^3*R^2*v^3*C^2-
16*X1^4*F^5*R^2*v^3*X*C+24*X1^4*F^6*R^2*v^2*X*C-16*X1^4*F^7*R^2*v*X*C-
2*X1^4*F^2*R^2*v^4*C^2+4*X1^4*F^4*R^2*v^4*X*C-4*R1^2*C^4*X1^2*F*v-
8*X^4*F^10*R1^2*X2*X1+16*X^4*F^9*R1^2*X2*X1*v-
4*F^5*R1^2*R^4*X1^2*v+2*F^4*R1^2*R^4*X1^2*v^2+8*F^3*R1^2*R^2*X1^2*C^2*v-
4*F^4*R1^2*R^2*X1^2*C^2-4*F^2*R1^2*R^2*X1^2*C^2*v^2-
8*X2^2*X1^2*X^3*F^10*C+4*X*F^4*R1^4*C*R^2-
16*X*F^5*R1^2*C*X1^2*R^2*v+8*X*F^4*R1^2*C*X1^2*R^2*v^2+8*X*F^6*R1^2*C*X1^2*R^2+1
6*X*F^3*R1^2*C^3*X1^2*v-8*X*F^4*R1^2*C^3*X1^2-
8*X*F^2*R1^2*C^3*X1^2*v^2+16*X1*X^4*F^7*C*v*R1^2+6*X^2*F^4*R1^4*C^2-
8*X2^2*X1^2*X^3*F^6*C*v^4+32*X2*X1*X^3*F^6*C*v^2*R1^2-
64*X2*X1*X^3*F^7*C*v*R1^2+32*X2^2*X1^2*X^3*F^7*C*v^3+32*X2^2*X1^2*X^3*F^9*C*v-
48*X2^2*X1^2*X^3*F^8*C*v^2-8*X1*X^4*F^8*C*R1^2-

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4*X^3*F^6*R1^4*C+64*X2*X1*F^5*R^2*v*X*R1^2*C-
32*X2*X1*F^7*R^2*v*X^2*R1^2+16*X2*X1*F^5*R^4*v*R1^2-
32*X2*X1*F^4*R^2*v^2*X*R1^2*C+16*X2*X1*F^6*R^2*v^2*X^2*R1^2-
8*X2*X1*F^4*R^4*v^2*R1^2+4*X1*R^2*C*v^2*X^2*F^4*R1^2+4*X1*R^4*C*v^2*F^2*R1^2-
4*X1*R^2*C^3*v^2*R1^2+32*X2*X1*C^3*v^2*X*F^2*R1^2-
48*X2*X1*C^2*v^2*X^2*F^4*R1^2+16*X2*X1*C^2*v^2*F^2*R1^2*R^2-
8*X2*X1*C^4*v^2*R1^2+8*X2^2*X1^2*F^8*R^2*X*C-
32*X2*X1*F^6*R^2*X*R1^2*C+16*X2*X1*F^8*R^2*X^2*R1^2-8*X2*X1*F^6*R^4*R1^2-
8*X1*R^2*C*F^5*v*X^2*R1^2-8*X1*R^4*C*F^3*v*R1^2+8*X1*R^2*C^3*F*v*R1^2-
8*R^2*C^2*F^3*v*X^2*R1^2+8*R^4*C*F^3*v*X2*R1^2-
16*X2^2*C^2*F^5*v*X^2*R1^2+16*X2^2*C^2*F^3*v*R1^2*R^2+4*R^2*C^2*F^4*X^2*R1^2-
4*R^4*C*F^4*X2*R1^2+4*R^2*C^2*v^2*X^2*F^2*R1^2-4*R^4*C*v^2*X2*F^2*R1^2-
8*X2^2*F^8*R^2*X^2*R1^2-8*X^4*F^5*C^2*v*R1^2-
16*X^4*F^7*C*v*X2*R1^2+4*X^4*F^4*C^2*v^2*R1^2+8*X^4*F^6*C*v^2*X2*R1^2-
4*X*C^2*F^2*R1^2*R^2-8*X*C^4*F^2*X2*R1^2+8*X2*F^3*R^4*v*R1^2-
16*X2^2*F^5*R^2*v*X*R1^2*C-
4*X2*F^2*R^4*v^2*R1^2+8*X2^2*F^4*R^2*v^2*X*R1^2*C+8*X2^2*F^10*C*X1^2*X^2+48*X2^2*
F^8*C*X1^2*X^2*v^2-8*X2*F^4*C*R1^2*R^2-
32*X2^2*F^9*C*X1^2*X^2*v+32*X2^2*F^7*C*X1^2*R^2*v-8*X2^2*F^8*C*X1^2*R^2-
16*X2^2*F^8*C^2*X1^2*X+64*X2^2*F^7*C^2*X1^2*X*v-96*X2^2*F^6*C^2*X1^2*X*v^2-
32*X2*F^7*C^2*X1^2*X^2*v+48*X2*F^6*C^2*X1^2*X^2*v^2+8*X2*F^4*C^3*X*R1^2+16*X2^2*F
^6*C^2*X*R1^2+8*X2^2*F^6*C*v^4*X1^2*X^2-8*X2*F^2*C*v^2*R1^2*R^2-
32*X2^2*F^7*C*v^3*X1^2*X^2+32*X2^2*F^5*C*v^3*X1^2*R^2-
48*X2^2*F^6*C*v^2*X1^2*R^2+64*X2^2*F^5*C^2*v^3*X1^2*X-16*X2^2*F^4*C^2*v^4*X1^2*X-
32*X2*F^5*C^2*v^3*X1^2*X^2+8*X2*F^4*C^2*v^4*X1^2*X^2+8*X2*F^2*C^3*v^2*X*R1^2-
8*X2^2*F^2*C^3*v^2*R1^2+16*X2^2*F^4*C^2*v^2*X*R1^2-
8*X1*F^3*R^4*v*R1^2+4*X1*F^2*R^4*v^2*R1^2+16*X2*F^3*C*v*R1^2*R^2-
16*X2*F^3*C^3*v*X*R1^2+16*X2^2*F^3*C^3*v*R1^2-
32*X2^2*F^5*C^2*v*X*R1^2+4*X2*C^2*F^2*R1^2*R^2-
8*X2^2*C^3*F^4*X*R1^2+8*X1*X^2*F^5*v*R1^2*R^2-4*X1*X^2*F^4*v^2*R1^2*R^2-
4*X1*X^2*F^6*R1^2*R^2+4*X2*X^2*F^6*R1^2*R^2+8*X2^2*X^2*F^6*R1^2*C^2-
8*X2^2*X^3*F^8*R1^2*C-
8*X2*X^2*F^5*v*R1^2*R^2+16*X2^2*X^3*F^7*v*R1^2*C+4*X2*X^2*F^4*v^2*R1^2*R^2-
8*X2^2*X^3*F^6*v^2*R1^2*C)^(1/2)-
2*X^2*X1^2*F^5*v+X^2*X1^2*F^6+X^2*X1^2*F^4*v^2+2*X*X1*C^2*v^2+2*X*X1*C^2*F^2-
4*X*X1*C^2*F*v-2*X2*X1*C^2*F^2+4*X2*X1*C^2*F*v-2*X1*R^2*C*F^2+4*X1*R^2*C*F*v-
2*X1*R^2*C*v^2-2*X2*X1*C^2*v^2+2*X2*X1*F^4*R^2-
4*X2*X1*F^3*R^2*v+2*X2*X1*F^2*R^2*v^2-2*X1*X^2*F^4*C-
8*X2*X1*X*F^3*C*v+4*X2*X1*X*F^2*C*v^2+4*X1*X^2*F^3*C*v-
2*X1*X^2*F^2*C*v^2+4*X2*X1*X*F^4*C-2*X*F^2*R1^2*C+X^2*F^4*R1^2-
F^2*R1^2*R^2+R1^2*C^2+2*X1^2*F^3*R^2*v-X1^2*F^2*R^2*v^2-X1^2*F^4*R^2-
2*X1^2*C^2*F*v+X1^2*C^2*F^2+X1^2*C^2*v^2-2*X1^2*X*F^2*C*v^2+4*X1^2*X*F^3*C*v-
2*X1^2*X*F^4*C-2*X2*X1*X^2*F^4*v^2-2*X2*X1*X^2*F^6+4*X2*X1*X^2*F^5*v)]
```

Appendix C

Oscillating Status of the Magnetic Inductance Roots

F	V	M1	M2
0.5	0.5	Re+Im	103.8333
0.51	0.5	-5.0438	1.9861
0.52	0.5	-3.0505	1.7419
0.53	0.5	-2.2244	1.4882
0.54	0.5	-1.7625	1.2712
0.55	0.5	-1.4643	1.0944
0.56	0.5	-1.2542	0.9511
0.57	0.5	-1.097	0.8338
0.58	0.5	-0.974	0.7364
0.59	0.5	-0.8744	0.6544
0.5	0.51	Re+Im	1.9715
0.51	0.51	Re+Im	-23.1761
0.52	0.51	-5.0184	2.0012
0.53	0.51	-3.0369	1.7532
0.54	0.51	-2.2153	1.4964
0.55	0.51	-1.7557	1.2773
0.56	0.51	-1.459	1.0991
0.57	0.51	-1.2498	0.9549
0.58	0.51	-1.0933	0.8368
0.59	0.51	-0.9709	0.7389
0.5	0.52	-10.2215	1.7204
0.51	0.52	Re+Im	1.9861
0.51999	0.52	Re+Im	2.1239
0.519999	0.52	Re+Im	2.1205
0.5199999	0.52	Re+Im	1.9345
0.51999999	0.52	Re+Im	65.2012
0.519999999	0.52	Re+Im	-375.5938
0.52	0.52	Re+Im	5.1771
0.52	0.52	Re+Im	11.5771
0.52000001	0.52	-6.48E+03	-2.5217
0.5200001	0.52	-2.05E+03	2.1519
0.520001	0.52	-647.9413	2.124
0.52001	0.52	-204.7065	2.1239
0.5201	0.52	-64.4168	2.124

0.521	0.52	-19.6927	2.1243
0.53	0.52	-4.994	2.0168
0.54	0.52	-3.0237	1.7649
0.55	0.52	-2.2064	1.5048
0.56	0.52	-1.7491	1.2835
0.57	0.52	-1.4537	1.104
0.58	0.52	-1.2455	0.9587
0.59	0.52	-1.0897	0.84
	:		
0.5	0.56	-1.8784	0.9308
0.51	0.56	-2.3132	1.0769
0.52	0.56	-3.0396	1.2595
0.53	0.56	-4.5472	1.4882
0.54	0.56	-10.537	1.7649
0.55	0.56	Re+Im	2.0498
0.56	0.56	Re+Im	-38.1992
0.57	0.56	-4.905	2.0853
0.58	0.56	-2.9743	1.8155
0.59	0.56	-2.1726	1.5412
0.5	0.57	-1.5852	0.8147
0.51	0.57	-1.8773	0.934
0.52	0.57	-2.3117	1.0811
0.53	0.57	-3.0379	1.2653
0.54	0.57	-4.5466	1.4964
0.55	0.57	-10.5745	1.7769
0.56	0.57	Re+Im	2.0673
0.569	0.57	Re+Im	2.2184
0.5699	0.57	Re+Im	2.2218
0.56999	0.57	Re+Im	2.222
0.569999	0.57	Re+Im	2.2234
0.5699999	0.57	Re+Im	1.8988
0.56999999	0.57	Re+Im	9.1362
0.569999999	0.57	Re+Im	13.5172
0.57	0.57	Re+Im	36.4875
0.570000001	0.57	-2.00E+04	0.7828
0.57000001	0.57	-6.32E+03	-24.562
0.5700001	0.57	-2.00E+03	2.0867
0.570001	0.57	-632.026	2.2183
0.57001	0.57	-199.674	2.2221

0.5701	0.57	-62.8311	2.2222
0.571	0.57	-19.2122	2.2227
0.58	0.57	-4.8848	2.104
0.59	0.57	-2.9628	1.8292
0.5	0.58	-1.373	0.7185
0.51	0.58	-1.5842	0.8172
0.52	0.58	-1.8758	0.9372
0.53	0.58	-2.3096	1.0854
0.54	0.58	-3.035	1.2712
0.55	0.58	-4.5433	1.5048
0.56	0.58	-10.5933	1.7893
0.57	0.58	Re+Im	2.0853
0.579	0.58	Re+Im	2.2402
0.5799	0.58	Re+Im	2.2438
0.57999	0.58	Re+Im	2.244
0.579999	0.58	Re+Im	2.2462
0.5799999	0.58	Re+Im	1.6976
0.57999999	0.58	Re+Im	-96.9102
0.579999999	0.58	Re+Im	-6.2257
0.58	0.58	Re+Im	52.0384
0.580000001	0.58	-1.99E+04	-38.125
0.58000001	0.58	-6.30E+03	-5.6338
0.5800001	0.58	-1.99E+03	1.9534
0.580001	0.58	-629.4382	2.2397
0.58001	0.58	-198.8555	2.244
0.5801	0.58	-62.5727	2.2442
0.581	0.58	-19.1329	2.2443
0.59	0.58	-4.8655	2.1234
0.5	0.59	-1.2111	0.6377
0.51	0.59	-1.372	0.7205
0.52	0.59	-1.5828	0.8198
0.53	0.59	-1.8738	0.9405
0.54	0.59	-2.3068	1.0899
0.55	0.59	-3.031	1.2773
0.56	0.59	-4.5374	1.535
0.57	0.59	-10.5928	1.8022
0.58	0.59	Re+Im	2.104
0.589	0.59	Re+Im	2.2629
0.5899	0.59	Re+Im	2.2666

0.58999	0.59	Re+Im	2.2668
0.589999	0.59	Re+Im	2.2627
0.5899999	0.59	Re+Im	1.2123
0.58999999	0.59	Re+Im	-14.3653
0.589999999	0.59	Re+Im	-2.1081
0.59	0.59	Re+Im	37.204
0.590000001	0.59	-1.98E+04	-55.362
0.59000001	0.59	-6.27E+03	-3.3119
0.5900001	0.59	-1.98E+03	2.319
0.590001	0.59	-627.0411	2.2682
0.59001	0.59	-198.0972	2.2668
0.5901	0.59	-62.3332	2.267
0.591	0.59	-19.059	2.2675
	:		
0.9	0.91227	-13.8715	3.23
0.9806	1	-4.3654	3.23
3.5	4.634	3.23	-0.2842
5	8.022	3.23	-0.3298
9	16.8035	3.23	-0.7856
18	35.25	3.23	-10.7039
36	71.3	3.23	-10.7038
72	142.975	3.23	-42.4297

Appendix D
Solutions of the Characteristic Equation
Of The No-load Self-excited Machine

Real part:

(1/C)-(((M*X1+M*X2+X1*X2)/(R1*R2*C))*(F-v)*F)=0

Imaginary part:

((M*F^2)/(R2*C))+((M+X1)*(F-v)*F/(R1*C))+((X2*F^2)/(R2*C))-(1/R2)=0

1.D Symbolic Solution For C & F :

C=
[(1/2/(M*X1+M*X2+X1*X2)*(M*X1*v+M*X2*v+X1*X2*v+(M^2*X1^2*v^2+2*M^2*X1*v^2*X2+2*M*X1^2*v^2*X2+M^2*X2^2*v^2+2*M*X2^2*v^2*X1+X1^2*X2^2*v^2+4*R1*R2*M*X1+4*R1*R2*M*X2+4*R1*R2*X1*X2)^(1/2))*M^2*X1*v+1/2/(M*X1+M*X2+X1*X2)*(M*X1*v+M*X2*v+X1*X2*v+(M^2*X1^2*v^2+2*M^2*X1*v^2*X2+2*M*X1^2*v^2*X2+M^2*X2^2*v^2+2*M*X2^2*v^2*X1+X1^2*X2^2*v^2+4*R1*R2*M*X1+4*R1*R2*M*X2+4*R1*R2*X1*X2)^(1/2))*M*X2^2*v+1/2/(M*X1+M*X2+X1*X2)*(M*X1*v+M*X2*v+X1*X2*v+(M^2*X1^2*v^2+2*M^2*X1*v^2*X2+2*M*X1^2*v^2*X2+M^2*X2^2*v^2+2*M*X2^2*v^2*X1+X1^2*X2^2*v^2+4*R1*R2*M*X1+4*R1*R2*M*X2+4*R1*R2*X1*X2)^(1/2))*X1*X2^2*v+1/(M*X1+M*X2+X1*X2)*(M*X1*v+M*X2*v+X1*X2*v+(M^2*X1^2*v^2+2*M^2*X1*v^2*X2+2*M*X1^2*v^2*X2+M^2*X2^2*v^2+2*M*X2^2*v^2*X1+X1^2*X2^2*v^2+4*R1*R2*M*X1+4*R1*R2*M*X2+4*R1*R2*X1*X2)^(1/2))*M*X1*v*X2+1/2/(M*X1+M*X2+X1*X2)*(M*X1*v+M*X2*v+X1*X2*v+(M^2*X1^2*v^2+2*M^2*X1*v^2*X2+2*M*X1^2*v^2*X2+M^2*X2^2*v^2+2*M*X2^2*v^2*X1+X1^2*X2^2*v^2+4*R1*R2*M*X1+4*R1*R2*M*X2+4*R1*R2*X1*X2)^(1/2))*M^2*X2*v+R1*R2*X2+R1*R2*M+R2^2*M+R2^2*X1)/(M*X1+M*X2+X1*X2)]

F=
[1/2/(M*X1+M*X2+X1*X2)*(M*X1*v+M*X2*v+X1*X2*v+(M^2*X1^2*v^2+2*M^2*X1*v^2*X2+2*M*X1^2*v^2*X2+M^2*X2^2*v^2+2*M*X2^2*v^2*X1+X1^2*X2^2*v^2+4*R1*R2*M*X1+4*R1*R2*M*X2+4*R1*R2*X1*X2)^(1/2))]

2.D Symbolic Solution For C & v:

C= -(C*R1*M+C*R1*X2-M^2*F^2*R1-F^2*R2*M^2-X2^2*F^2*R1-R1*R2^2-2*M*F^2*R1*X2)/F/R2/M^2

v= -M*(-M*X2*F^2-X2^2*F^2+X2*C-R2^2)/(-M^2*F^2-2*M*X2*F^2+M*C-X2^2*F^2+X2*C-R2^2)

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